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DYNAMO DESIGN.

ITS

THEORY AND PRACTICE.

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PREFACE.

The following pages presuppose a practical knowledge of the theory of electricity and magnetism. In this wise the book under notice can only serve supplementally to the many good treatises already dealing with the subject. The table of contents will show that it was the aim to furnish theoretical grounds for the choice of dynamo constants. Considerable matter will be found treated of for the first time, in a book of this compass, and it may be stated in passing that no excuse need be offered for employing the calculus notation. In the opinion of the writer this latter method is a *sine qua non* for all successful engineering work. Before actual examples in design are undertaken, a set of simple formulæ are evolved for determining the over-all dimensions of the carcase and the dynamo losses. The iron and copper losses of the armature are made to depend upon the slot dimensions, and for trial variations of the latter, tables are constructed showing the advantages or disadvantages of successive choices. It is by referring all losses to the unit peripheral surface of the net armature core that, it would seem, a better grasp of the relative loss weights is obtained. Much has been said about the incremental loss at full load. The evidence is that it is an iron loss and in the following a practical method of evaluation of this latter has been set forth. In an example an application of this formula is made and the influence upon the choice of air gap noted. The effect of the time load-factor upon the final choice of air gap is also shown. These latter are entirely novel views and it is hoped they may prove useful.

THE AUTHOR.

New York, September, 1905.

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CHAP. XVII. Examples of Dynamo Computation. Law of Cost of Dynamos. Determination of the Number of Poles. Field Frame Magnetic Densities.

LIST OF SYMBOLS.

A = Cross section of annulus of field coil.
AB = Ampere-bars.
$\mathbf{A}$ = Ampere-turns.
B = Magnetic density.
B_a = Apparent magnetic density in teeth.
B_r = Real magnetic density.
c = Specific armature copper loss.
D = Diameter of armature.
D_c = Diameter of commutator.
E = Voltage.
e = Specific excitation loss. Sparking voltage.
e_l = Sparking voltage with lap winding.
e_w = Sparking voltage with wave winding.
F = Flux per pole.
f = Pole-pitch factor.
i = Specific iron loss.
i_s = Current density net in armature copper.
J = Current in amperes.
L = Axial end connection length of armature winding. Self-induction.
$L.M.T.$ = Length of mean turn.
l = Length of armature core.
$m = \frac{B_a}{B_r}$.
n = Specific iron loss increment. Number of armature slots.

p = Number of poles.
R = Revolutions per minute.
r = Tooth pitch ratio = $\frac{\text{tooth width}}{\text{tooth width} + \text{slot width}}$
S = Design coefficient.
s = Ratio of diameter at bottom of slots to diameter of armature.
T = Time constant. Time of commutation.
t = Specific teeth iron loss. Time. Turns per coil.
V = Volume.
v = Ventilating factor.
W = Watts output or input.
z = Tooth pitch.
$\sim$ = Frequency in cycles per second.
Δ = Current density (gross) in field copper.
Δ_b = Current density in brush per cm^2.
δ = Depth of air gap.
$\varepsilon = \frac{D}{p\,l}$.
η = Efficiency
$\kappa = D^2\,l$.
Λ = Length of field coil mean turn.
λ = Depth of armature slot.
ν = Number of commutator parts.
ρ = Ratio of depth of slot to width of slot.
σ = Area of brushes.
τ = Pole pitch. Temperature.
ϕ = Copper factor.
φ = Specific resistance of copper per cm^3.

CHAPTER I.

THE DETERMINATION OF DIAMETER AND LENGTH OF CORE.

In whatever manner the transformation of energy in the higher forms takes place, a certain amount must be degraded into heat. Friction, whether electrical or mechanical, is ever present, and since the capacity for heat of a body is limited, it is evident that the temperature will rise as more and more energy is wasted. In the dynamo one has the usual copper losses due to the armature and field windings, the bearing friction and windage loss, the loss of the brushes due to carrying current and the brush friction loss. All these serve to diminish the efficiency of transformation of energy in the dynamo or motor. Since, therefore, such losses must inevitably occur in greater or lesser degree, a means must be provided by so arranging the radiative surfaces that the temperature rise shall not exceed a certain figure. This latter figure is the limit above which the wasteful deterioration of materials would take place.

The engineer is thus confronted with two problems: to design the machine that the efficiency shall be as high as possible; and to endeavor to attain the permissible temperature rise since a lower temperature will mean a lack of economy of materials by providing a large volume and hence a correspondingly great radiating surface.

The radiative capacity of an open type dynamo or motor armature is known to depend upon the peripheral speed and upon the cylindrical surface of the entire armature windings. The curve, Fig. 1, shows this relation for a permissible temperature rise of 40° C.

From previous tested designs the possible efficiency for a given size and output is fairly well known. Hence,

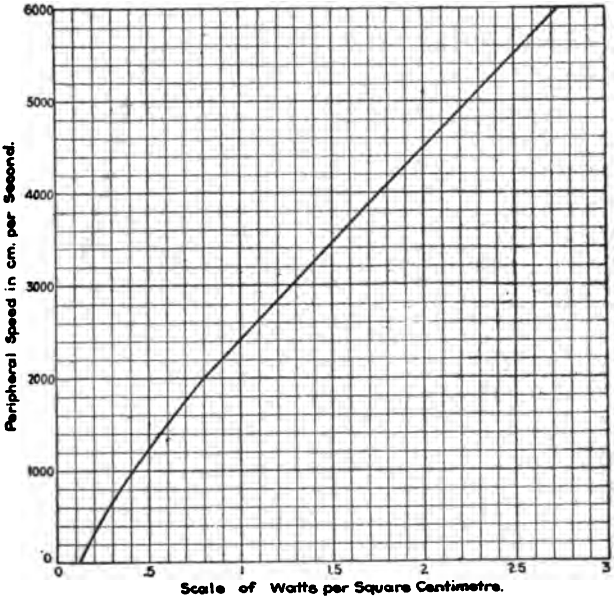

FIG. 1.—Curve giving Armature Radiative Capacity.

if a given specification determines the output and speed, it should be possible to quickly settle upon the relation of the armature to the overall dimensions that will be necessary in order that the condition as to temperature rise shall obtain.

It will be observed from Fig. (1) that the watts per square centimetre that can be radiated per unit of peripheral speed is practically a constant quantity. Let this be called the radiating constant. Then let

η = efficiency.
τ = final temperature rise.
D = diameter of armature.
L = length axially of the armature end connections.
l = length of core (including ventilating spacings).
W = output in watts.
ν = radiating constant for temperature rise τ_0.
$100\, f_p$ = factor of total loss due to energy transformation giving loss radiated by armature.

Then since $\frac{W(100-\eta)}{100}$ = loss in watts of apparatus,

$$W f_p (100-\eta) = \text{armature watts dissipated.} \qquad (1)$$

By means of Fig. (1)

$$\frac{\text{Watts radiated}}{\text{surface} \times \text{peripheral speed}} = \frac{\nu \tau}{\tau_0}. \qquad (2)$$

Hence combining (1) and (2)

$$W f_p (100-\eta) = \frac{\nu \tau}{\tau_0} \times \pi D (L+l) \times \pi D \text{ revs.}$$

and

$$W = \frac{\pi^2 \nu \frac{\tau}{\tau_0}}{f_p (100-\eta)} . D^2 (L+l) \text{ revs.} \qquad (3)$$

This is the most general form of output equation. It is usual for constructional reasons to take the length of the core as proportional to the pole pitch. (The pole-pitch is the periphery of the armature divided by the number of poles.) Usually, the aim is a square pole-shoe, so as to obtain the minimum condition of round poles with a view to the saving of field copper; but the end connections are found in practice to be also proportional to the pole pitch, since the angle subtended by the ends is in the neighborhood of 120° for all cases. This latter condition, therefore, permits of a minimum armature copper loss for a ratio of D to l in terms of p that practically coincides with the condition of square pole-shoes.

The armature winding length of mean turn is from Fig. (2), twice the armature length, or $2\,l$, and twice the end connection length, or $2\,(A\,C\,B)$. Since $A\,C\,B$ forms an isosceles triangle of 120° whose base is $\frac{\pi\,D}{p}$,

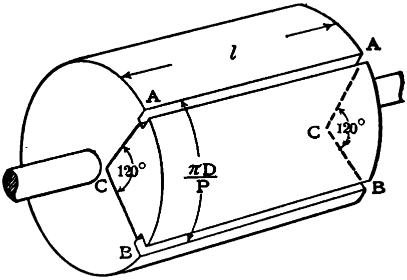

FIG. 2.—Showing Position of Armature Coil.

$$\text{length of mean turn} = 2\,l + \frac{c\,\pi\,D}{p}, \qquad (4)$$

when $c = \frac{4}{\sqrt{3}}$.

If the $D^2\,l$, which is roughly proportional to the volume of the machine, is taken as a constant; say

$$D^2\,l = \kappa \qquad (5)$$

the expression (4) can be minimized with respect to D

by taking $\frac{d}{dD} = 0$. Using the expression (5),

$$\frac{d}{dD}\left(2\,l + \frac{c\,\pi\,D}{p}\right) = 0 \text{ gives}$$

$$\frac{D}{l} = \frac{4}{c\,\pi\,p} = {}^{\cdot}55\,p = \varepsilon\,p \qquad (6)$$

Obviously a small length of mean turn will mean less copper resistance, and hence less copper loss.

Let the pole-shoe multiplier, f, (ratio of pole-shoe span to pole-pitch) be .7, then, for square pole-shoes the length of core equals $\frac{.7\,\pi\,D}{p}$, and

$$\frac{D}{l} = .455\,p. \qquad (7)$$

This is very nearly the value given by equation (6). The value of L can be evaluated in terms of D and p, (it is twice the altitude of the isosceles triangle $A\,C\,B$), and then the formula for output becomes

$$W = \frac{\pi^2\left(1 + \frac{\pi\,\varepsilon}{\sqrt{3}}\right)\nu\,\frac{\tau}{\tau_0}}{f_p\,(100 - \eta)}\,.D^2\,l \text{ revs.}$$

$$= S\,.D^2\,l \text{ revs.} \qquad (8)$$

The quantity S may be called the " design coefficient," and is seen to have a definite physical meaning. *It is proportional to the temperature rise and the specific radiative capacity and inversely proportional to the factor giving the armature loss, and the entire loss of efficiency of the machine.*

Fig. (3) gives the value of S for different value of W taken from practice. One can, however, use the following empirical formula which is based upon Fig. (3), viz.:

$$D^2 l.\ \text{revs.} = 36\times10^4 (1.5+\sqrt{W\times10^{-3}})^2 \qquad (9)$$

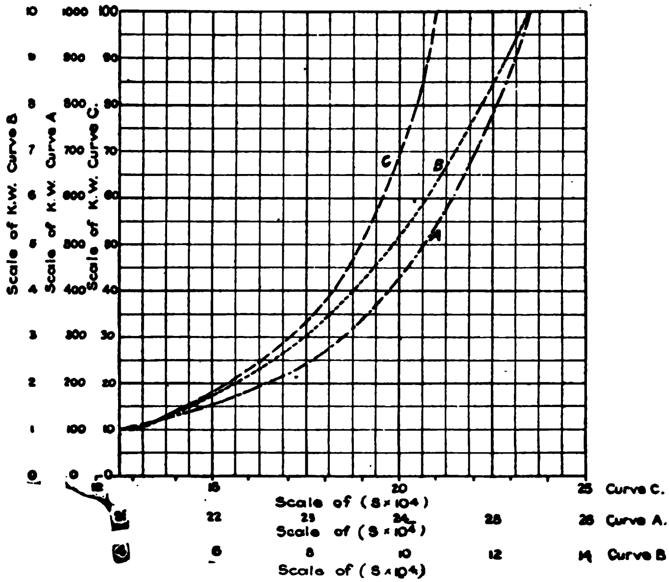

FIG. 3.—Output Design Coefficient Curves.

This latter equation holds good for from 1–1000 kw. It may be remarked in passing that if in equation (8) the value f_p is taken as a constant, a formula for the efficiency based upon (8) and (9) gives

$$\eta = 100 - 4.5\left(\frac{3.5}{\sqrt{W\times10^{-3}}} + 1\right)^2 \qquad (10)$$

It should also be noted, that with formulas (6) *and* (9) *the dimensions D and l of the armature follow as soon as the value for p is determined.* The value of p chosen will affect the whole course of design of the machine. In a measure it will affect the efficiency since the frequency of magnetization and demagnetization of the armature iron will be dependent upon the revs. and the number of poles. It will also affect the commutation, and lastly, though not least, it will affect the element of price. This latter will be discussed below. It would appear that different values of p should be chosen so that one can weigh the relative merits and demerits of a particular design. An empirical rule involving p is the following:

$$2^{\frac{p-4}{2}} = \frac{D^2 l \, (c\,m^3)}{150.000}$$

Strictly speaking, the value of f_p in equation (8) is not a constant. Formula (10) has value in giving one an approximate idea of the real efficiency to be expected for a given output.

CHAPTER II.

THE DETERMINATION OF THE POLE PITCH FACTOR f.

The pole-pitch multiplier f will affect the character of the flux distribution curve with respect to the periphery of the armature. Let Fig. (4) be a sine distribution of magnetic density taken at such periphery,

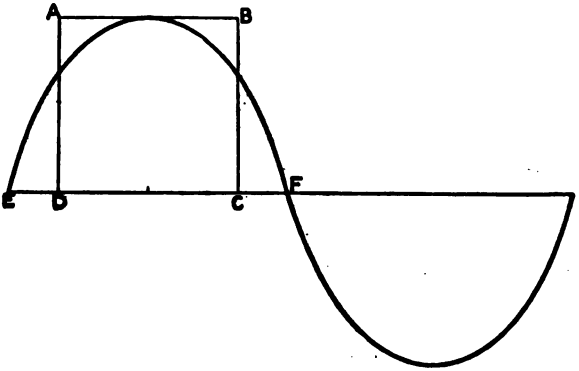

FIG. 4.—Armature Flux Distribution Curve.

and let area $A\,B\,C\,D$ equal the area of the sine curve $E\,A\,B\,F$. The pole-pitch multiplier will be $\frac{D\,C}{D\,F}$, since with ordinary pole-shoe constructions the density at the base of the pole-shoe may be considered uniform. In a sine curve the ratio $\frac{D\,C}{E\,F}$ is $\frac{2}{\pi} = .636$. With other curves the pole-shoe span will give other values for f. The particular flux distribution curve can be obtained by connecting a coil encircling a tooth with an oscillograph. Such a curve would present a graphic view of the state of magnetization taking place in the armature teeth during the cycle of operations caused by rotating the armature in a magnetic field. For any cycle of magnetic operations there will, in general, be two types of losses: the hysteresis loss, depending upon the maximum density of an alternation and the number of cycles; and the eddy current loss, depending upon the rate of change of magnetic density squared, when the material and flux density are homogeneous throughout. The eddy current loss is that due to parasitic currents set up in the material by the varying flux, and since the watts are proportional to $\int_0^t e^2\,d\,t$, it follows because of e, the voltage of a parasitic circuit, that the loss depends upon the rate of change of flux squared at any instant of time t. The resistance of a parasitic circuit is assumed constant. The hysteresis loss is due to a sort of magnetic friction. Curves giving the iron loss per $(\text{cm})^3$ of iron are shown in Fig. (5).

Since the eddy current losses in dynamos form quite an important item of the total losses of a machine, it will be of value to determine the particular shape the flux distribution curve should take, in order that the loss may be reduced to a minimum. Further, the curve when once found will enable the particular pole-shoe span to be determined. In other words the most economical value of f will have been found, and the shaping of the pole-shoe horns would then come up for consideration. The definite integral $\int_{t_0}^{t_1} \left(\frac{d\,B}{dt}\right)^2 d\,t$ will measure the eddy current loss in the teeth as the teeth successively pass through the magnetic field, no matter what may be the flux distribution at the periphery. It will be assumed in the following that the magnetization of the teeth individually is uniform. With then, a given curve area, or flux, it will be necessary to minimize the expression $\int_{t_0}^{t_1} \left(\frac{d\,B}{d\,t}\right)^2 d\,t$ subject to the condition

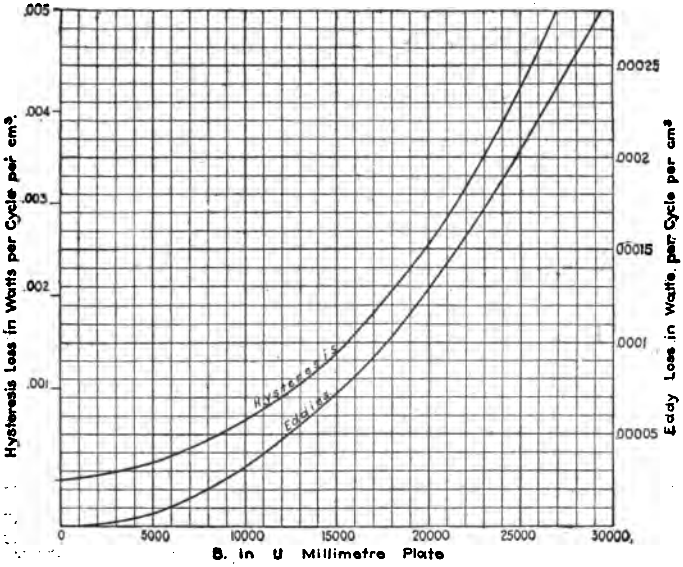

FIG. 5.—Armature Iron Loss Curves.

that $\int_{t_0}^{t_1} B\,d\,t$ equals A, that is, a constant. The minimum function is a parabola of the form $y = \frac{4\,h}{b}x - \frac{4\,h}{b^2}x^2$, when h = the height or maximum density, and b is the base or pole-pitch. It is well known, however, that the curve of sines very closely approximates to the parabola, and hence, so far as eddy loss is concerned, the curve of distribution may be taken as the sine or parabolic curve. The value of f for the parabola is $\frac{2}{3} = .666$, and hence $f = .7$ (for a round number) would constitute very good practice. In the above no account was taken of hysteresis. In actual working, one can take account of hysteresis, by determining such a flat top wave with parabolic sides, as will give a minimum total iron loss for a given flux, and given polar span. In a parabola, the eddy loss is proportional to $\frac{h^2}{b}$. See Fig. (6). In the rectangle, as seen, the eddy loss is zero. Then let reference be made as to

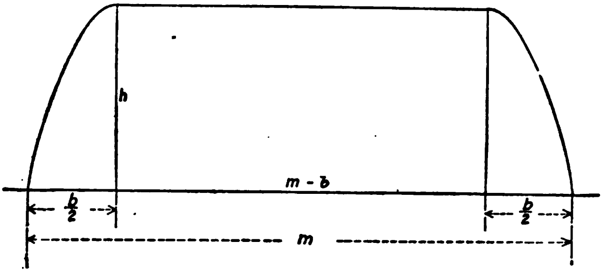

FIG. 6.—Minimum Iron Loss Flux Distribution Curve.

loss, to unit peripheral speed with parabola with base m, and having unit height. The base m will thus be fixed, and the area of the figure may be taken as A, a constant. The eddies will thus increase inversely proportional to the base length taken for the parabola, the rectangular area interposed having no iron loss. The hysteresis loss will be proportional to h^2 and would only alter if h altered. [The index is taken as the square though it might be anything from 1.4 to 2.3.] Total loss = $\frac{m\,h^2\,s}{b} + h^2\,t$ where s and t are constants. The area of the figure will be

$$A = h\,(m - b) + \tfrac{2}{3}\,b\,h.$$

Hence work done per half cycle

$$= \frac{9\,A^2\,(m\,s + b\,t)}{b\,(3\,m - b)^2}$$

This expression will be a minimum for $\frac{d}{d\,b} = 0$, which gives

$$b = \frac{(24\,m^2\,t\,s + 9\,m^2\,s^2)^{\frac{1}{2}} - 3\,m\,s}{4\,t}$$

The general expression for b shows, that with given flux, and given polar embrace, as the hysteresis becomes relatively more and more important, that is, on low speed machines, waves approaching a flat top should be sought, whereas where the eddies exert a preponderating, or, at least a commensurable influence, this is in high speed machines, such as are now in common use, a wave approaching the parabola or sine would be of advantage. There are, however, other reasons* to show why the pole-pitch factor, f, should be chosen in the neighborhood of .7. One is, that the distortion due to the field set up by the armature ampere turns will be relatively small for $f = .7$. Professor Arnold has calculated a set of curves showing the form of the armature field distribution for different values of f. An inspection of these curves will determine at once the economy of choosing an approximate sine wave of distribution.

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*See under R. V. sparking constant, p. 45.

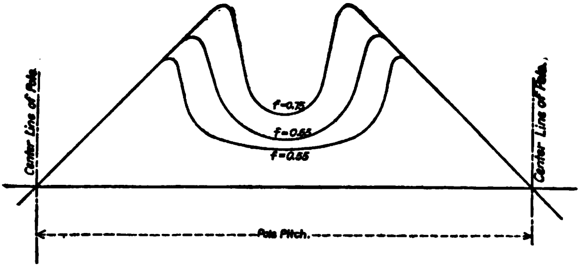

Fig. 7.—Armature Cross Flux Distribution Curves.

It may be well to add that the apparent discrepancy between so-called theory and practice in the matter of eddy-current loss is almost entirely due to a neglect of the effect of flux distribution in calculating out these losses.*

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*See papers by author in *The Electrician* (London), Oct. 16, 1903, and *Elec. World and Engineer*, (New York), April 8, 1905

CHAPTER III.

ARMATURE IRON AND COPPER LOSS.

The flux per pole is given by the polar area of the teeth in a pole-pitch, multiplied by the pole-pitch factor, and multiplied further by the apparent density in the teeth. If $B_r =$ real density in the teeth the apparent density $= B_r + H_r \left(\frac{1 - v\,i\,r}{vir}\right) = B_a$.* The flux per pole is thus $\frac{\pi\,D}{p} f\,B_a\,l\,i\,v$. Since the current per circuit is equal to $J = \frac{W}{p\,E}$ and since by the voltage equation $E = \frac{\text{Bars} \times R \times F}{60 \times 10^8}$, the ampere bars, AB, follow by elimination. Thus

$$\text{AB} = \frac{60 \times 10^8 \times W}{p.R.F.}$$

It will simplify matters to refer losses to unit peripheral length, and unit net length of l axially of the armature core.

---

*The development of the formula is gone into in the Formulary, see page 69.

The AB per unit length of periphery of the armature are

$\text{AB} = \dfrac{60 \times 10^8 \times W}{\pi D R p F}$ This latter may be called the *specific current volume* of the armature. The length of actual conductor taking into account the end connections will be given by $l\left(1 + \dfrac{2\,\pi\,\varepsilon}{\sqrt{3}}\right)$* The slot cross section available for winding will be $\pi D\,(1 - r)\,\lambda$ and the specific cross section will therefore be

$$\frac{\pi D\,(1 - r)\,\lambda}{\pi D} = \lambda\,(1 - r).$$

If ψ is the specific resistivity of copper at the temperature of 40° C. and ϕ is the factor of the space available giving the net copper, we have

$$\text{Specific copper loss} = \left(\frac{60 \times 10^8 \times W}{\pi^2 D^2 l f B_a riv R}\right)^2 \frac{\psi}{\phi} \; \frac{\left(1 + \dfrac{2\,\pi\,\varepsilon}{\sqrt{3}}\right)}{iv\,(1 - r)\,\lambda}$$

$$= \left(\frac{60 \times 10^8}{\pi^2 f}\right)^2 \left(1 + \frac{2\,\pi\,\varepsilon}{\sqrt{3}}\right) \frac{\psi}{\phi} \cdot \frac{S^2}{(1 - r)\,\lambda\, r^2\, B^2_a\,(i\,v)^3}$$

The teeth iron loss is given by†

Teeth loss $= \alpha\, B^2 \sim^x \times$ volume of teeth.

---

*See under output equation page 4.

†See also page 77 under same heading.

The volume of teeth depends upon the depth of slot λ, and r the tooth pitch ratio. The total volume of teeth is $\pi D r l i v$, and thus the specific volume of teeth, *i.e.*, per unit of periphery and unit net length of core, is

$$\frac{\pi D r l i v \lambda}{\pi D l i v} = r \lambda.$$

The value of x is 1.4 and $\alpha = 4.14 \times 10^{-4}$ when $\frac{B}{10.000}$ is taken instead of B. Thus

Specific iron loss in teeth $= 4.14 \times 10^{-4} \left(\frac{B}{10.000}\right)^{2} \sim {}^{1.4} r \lambda.$

For determining the combined core and teeth iron loss, the combined loss can be expressed as a factor of the teeth iron loss. One has

Flux per pole $= \frac{\pi D}{p} B_a r l i v$, and thus the core density must necessarily be $\frac{\pi D}{p} B_a \frac{r l i v}{\text{Double depth of core}}$.

By Dr. Thornton's rule* the most profitable employment of the core iron will take place when the double depth of core proper is to the diameter of the core as $\frac{2}{p}$. If greater depths than that indicated by Thornton's rule are employed, a rapid falling off in the magnetic density takes place. Thus, if the diameter at bottom of slots is $s D$, where s is a multiplier, and double depth of core is d, then

$$\frac{d}{D s} = \frac{2}{p}$$

---

*See paper delivered before British Asscciation, London, 1904.

Hence $d = \frac{2\,D\,s}{p}$ and the density in the core (average) $= \frac{\pi\,B_a\,r}{2\,s}$.

The volume of the core $= \frac{\pi}{4}\left\{D^2\,s^2 - (D\,s - d)^2\right\} l\,i\,v$

$$= \frac{\pi}{4}\,d\,(2\,D\,s - d)\,l\,i\,v,$$

but by Thornton's rule

$$d = \frac{2\,D\,s}{p},$$

and, therefore, volume of core

$$\frac{\pi}{4}\cdot\frac{2\,D\,s}{p}\left(2\,D\,s - \frac{2\,D\,s}{p}\right) l\,i\,v = \frac{\pi\,D^2\,s^2}{p}\left(1 - \frac{1}{p}\right) l\,i\,v.$$

The specific volume will be, since $\frac{D}{p} = \varepsilon\,l$,

$$\text{specific core volume} = \frac{\pi\,D\,s^2\left(1 - \frac{1}{p}\right).\,\varepsilon\,l^2\,i\,v}{\pi\,D\,l\,i\,v}$$

$$= s^2\left(1 - \frac{1}{p}\right)\varepsilon\,l$$

and specific core iron loss

$$= \frac{\alpha\,\pi^2\,\varepsilon\left(1 - \frac{1}{p}\right) B^2_a\,r^2 \sim^x l}{4}$$

Thus taking $B_a = m\ B$ we have *total specific teeth and core iron loss = teeth loss times*

$$\left(1 + \frac{\alpha\,\pi^2\,\varepsilon\left(1 - \frac{1}{p}\right) B^2{}_a\, r^2 \sim^x l}{4\,\alpha\, B^2 \sim^x \lambda\, r}\right)$$

or, *teeth loss* $\left(1 + \frac{\pi^2\,\varepsilon\, m^2}{4}\,.\, r\left(1 - \frac{1}{p}\right).\,\frac{l}{\lambda}\right)$

CHAPTER IV.

DETERMINATION OF THE WORKING DENSITY IN COPPER AND IRON.

It was seen in the chapter on Iron and Copper Loss that the total ampere-bars, AB, was given by the relation

$$\text{AB} = \frac{60 \times 10^8 \times W}{p.R.F.}$$

The total available slot cross section is equal to $\pi D \lambda (1-r)$. Hence since the copper factor is ϕ, the net current density i_s in the armature copper is

$$i_s = \frac{60 \times 10^8 \times W}{p.R.F.\phi.\pi D \lambda (1-r)}$$

$$\text{Since however } F = \frac{\pi D}{p} l\, i\, v\, B_a f$$

$$i_s = \frac{60 \times 10^8 \times W}{R \pi D l\, i\, v\, B_a f \times \pi D \lambda (1-r)},$$

$$\text{but } \frac{W}{D^2\, l.R.} = S. \quad \text{Hence}$$

$$i_s = \frac{60}{\pi^2\, i\, v\, m\, f} \times \frac{S \times 10^4}{\lambda \left(\frac{B}{10.000}\right)} (1-r)$$

Here it is taken that $B_a = m\,B$ where B is the net magnetic density in the armature teeth and B_a is the apparent density on the assumption that all the flux passes from the air gap into the teeth and none by way of the air path in the slots. It can be taken that the current density in the armature copper will be 75 amperes per square centimeter. With this assumption, a tentative value can be obtained for the magnetic density in the armature teeth. The value of m can be assumed to be in the neighborhood of 1.05 and the value of r as .5. The magnetic density will then be expressed in terms of λ and S. The particular density to work with will depend upon the machine itself, but this will be gone into later. Making the substitution indicated, the formula results in

$$B = 3200 \times \frac{S \times 10^4}{\lambda}$$

CHAPTER V.

SLOT SHAPE DETERMINATION.

It was seen that the working density in the teeth was made to depend upon the design coefficient and the depth of slot. In designing the slots of an armature the starting point can be made the depth of the slots. It is obvious that increasing the depth will tend to increase the copper volume on the armature. On the other hand too small a value will mean too great a density in the air gap and a large excitation loss. The following empirical formula for the depth of slots will be found to give good results, but a modification in the depth can be resorted to during the course of the design. Let

$$\lambda = 2 \log_{10} D \text{ (cm.)}$$

Settling upon a given depth of slot, different values in succession should be tried for the value of r. The formula for the specific iron and copper losses have been developed with a view to facilitating the choice of the best value of r. The formulas are quite simple in character and examples of their use will be given later on. The specific radiative capability of the armature should also be determined and in general it will be found that going beyond some value of r would result in overheating of the armature body. On the other hand, some smaller value of r will give a more economical result from the point of view of efficiency, but on the other hand, the copper volume and hence the copper density will be diminished. The final choice of r will therefore depend upon the relative importance of efficiency to heating. This must of necessity be left to the designer. The specific excitation loss should therefore also be tabulated. (See examples given further.) And finally the current density should be gone into, In starting the tabulation a given density is assumed, preferably that given by the formula for density derived from i_s. Then the iron and copper losses are calculated for the different values of r. (Take values of r equal to .4, .5, .6, .7, etc., respectively.) The total losses for the iron and copper of the armature are then found for the value of B assumed. However, in general some other value of B will give better results for particular values of r and λ. The determination of this value is arrived at in a manner depending upon the following: Firstly, it will be necessary to show that that density will be the most efficient for which the iron loss of the armature equals the copper loss. With this condition the best value of B will easily follow. The iron loss depends upon B^2 and the copper loss upon $\frac{1}{B^2}$. This results from inspection of the formulas for the specific losses. Let i = iron loss and c = copper loss.

$$i = a\,B^2 \text{ and } c = \frac{b}{B^2}.$$

$$\text{Total of } i + c = a\,B^2 + \frac{b}{B^2}.$$

$$\text{Taking } \frac{d}{d\,B} = 0$$

$$2\,a\,B = \frac{2b}{B^2} \text{ or } a\,B^2 = \frac{b}{B^2}.$$

Hence the iron loss should equal the copper loss for the most efficient value of B provided r and λ are kept constant.

Assume that B_1 is the value of the density taken in determining the iron and copper losses, given by $a\,B^2{}_1$ and $\frac{b}{B_1{}^2}$ respectively, what shall be the relation of the new value of B to the old value B_1 to give minimum iron and copper losses? Let

$$a\,B_1{}^2 = x\,.\,\frac{b}{B_1{}^2}.$$

Substituting the new value of B, the losses will be $a\,B^2$, and $\frac{b}{B^2}$. But now these losses are equal to each other. Thus

$$a\;B^2 = \frac{b}{B^2} \text{ and } a\,B_1{}^2 = x\,\frac{b}{B_1{}^2}$$

$$\frac{a}{b} = \frac{1}{B^4} = \frac{x}{B_1{}^4}. \text{ Hence } B = \frac{B_1}{x^{1/4}}.$$

Thus the new value of the density is determined *by taking the old density, and dividing it by the fourth root of the division of the old iron loss by the old copper loss.*

To determine the minimum combined iron and copper losses the result can be obtained by considering the value of x.

$$x = \frac{i}{c}, \text{ hence, } \sqrt{x} = \sqrt{\frac{i}{c}}$$

The old iron loss is $a\,B_1{}^2 = i^1$; the new is $a\,B^2 = i$. Hence

$$\text{New iron loss} = a\,B_1{}^2 \times \left(\frac{B}{B^1}\right)^2$$

$$\text{but } \frac{B}{B^1} = \frac{1}{x^{\frac{1}{4}}}. \quad \text{Hence,}$$

$$\text{new iron loss} = \text{old iron loss} \times \frac{1}{\sqrt{x}}$$

$$\text{If } i = \text{old iron loss and } x = \frac{i}{c}$$

$$\text{new iron loss} = i \times \frac{1}{\sqrt{\frac{i}{c}}} = \sqrt{i\,c} = \text{new copper loss.}$$

Hence total minimum iron and copper loss equals twice the square root of the iron loss times the copper loss. This theorem will be used in the course of the designs given later.

CHAPTER VI.

IRON LOSS INCREMENT ON FULL LOAD.

It has been pointed out that the form of the flux distribution curve exercises a marked influence upon the magnitude of the eddy current losses in the teeth. It was for this reason that the most economical value of *f* was settled upon as .7. However, when the armature takes on load the ampere bars of the armature set up a field of flux at right angles to the polar flux, and the two fluxes combining, there results a distorted field. Fig. 8 gives a graphic view of an actual flux distribution curve due to the armature ampere turns. Were it not for the interpolar air spaces the distribution would be strictly triangular as indicated by the dotted lines; but since the pole-shoe span is discontinuous with respect to the armature periph-

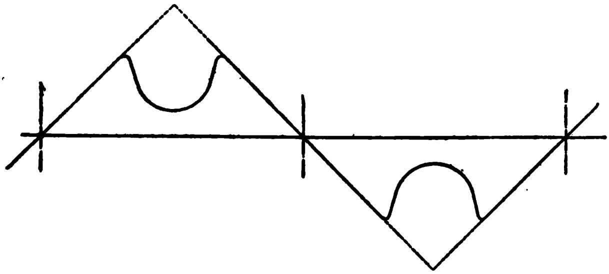

FIG. 8.—Armature Cross Field Curves.

ery, the true distribution is as shown by the full line. The ordinary forms of pole-shoes are of course referred to. It is obvious that the armature body revolving in this distorted field will result in the eddy current losses being increased to conform with the new condition. It will be the object of the following to firstly show that the two fields can be considered separately, that is, the eddy losses can be calculated for the two fields independently and then added; and secondly, to obtain a formula from which the magnitude of the increased loss can be estimated. The open circuit (no load) flux distribution curve will be taken as a sine wave, and for the armature cross field the representation will be as shown in Fig. 9, in which the true curve is approximated to by a series of straight lines. A calculation of the $\int_0^\pi \left(\frac{dB}{dt}\right)^2 dt$* of an actual cross-

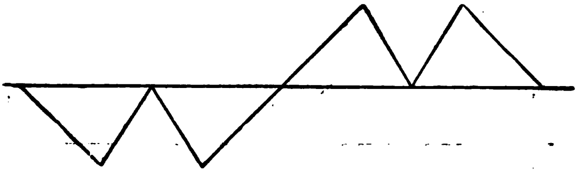

FIG. 9.—Approximated Armature Cross Field Curve.

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*It might be worth while to dignify the expression $\frac{\int_0^\pi \left(\frac{dB}{dt}\right)^2 dt}{\int_0^\pi B\,dt}$ as the "loss factor" of a given flux distribution curve.

flux curve will show how near such an approximation outlined above is. Further, an empirical rule deduced from actual experiment agrees quite well in character with the formula developed in the sequel. In Fig. 10 are drawn two curves, the one a sine curve whose equation is

$$y = B \sin x$$

where B is the maximum induction, and x is measured along the armature periphery, and $y' = \beta \frac{2x}{\pi} - \beta$ is the equation for the part of the cross flux curve between a and b, the limits of the pole-shoe arc and the position of the pole-shoe horn tips. β is the maximum density due to the armature ampere turns, with the assumption of a continuous peripheral pole-shoe, and thus, when $y = 0$, $x = \frac{\pi}{2}$ (or one has the midpole condition), and when $y' = \beta$, $x = 0$ or π. Adding the two ordinates, that is, superimposing the two flux distribution curves, the equation of the combined curves is

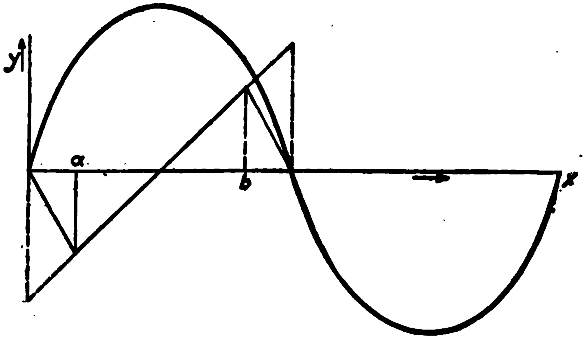

FIG. 10.—Armature Flux Distribution Curves.

$$Y = B \sin x + \frac{2x\beta}{\pi} - \beta.$$

Assuming that it is sufficiently exact to measure the eddy loss by

$$\int_a^b \left(\frac{d\,Y}{d\,x}\right)^2 d\,x \text{ we have}$$

$$\text{Eddy loss}\Big|_a^b \equiv \int_a^b \left(B^2 \cos^2 x + \frac{4\,\beta^2}{\pi^2} + \frac{2\,\beta\,B \cos x}{\pi}\right) d\,x$$

$$= B^2\left(\frac{x}{2} + \frac{\sin 2\,x}{4}\right) + \frac{4\,\beta^2\,x}{\pi^2} + \frac{2\beta \sin x}{\pi}\;B\;\Big|_a^b$$

This expression shows immediately that the same loss would be obtained had one calculated the losses of the fields separately and then added, since the term $\frac{2\,\beta\,B\,\sin\,x}{\pi}\Big|_a^b$ goes out to zero for all symmetrical fields. Thus at least this part of the losses superimpose. It will be now necessary to consider the fields in the interpolar region. Using the same figure but taking the case of the straight line between $x = 0$ and $x = a$, the straight line equation is $y' = -\frac{\beta}{a}x$ and, as before, $y = B \sin x$. Adding ordinates

$$Y' = B \sin x - \frac{\beta}{a}\,x \text{ and the eddy loss}\Big|_o^a \quad \int_o^a \left(\frac{d\,Y'}{dx}\right)^2 d\,x$$

$$= \left| B^2\left(\frac{x}{2} + \frac{\sin 2x}{4}\right) + \frac{\beta^2 x}{a^2} - \frac{2B\beta \sin x}{a} \right|_o^a$$

Investigating now from $x = \pi - a$ to $x = \pi$ where

$$y = B \sin x$$

$$y' = -\frac{\beta}{a}x + \frac{\pi}{a}\beta, \text{ and}$$

$$Y'' = B \sin x - \frac{\beta}{a}x + \frac{\pi}{a}\beta$$

$$\int_{\pi - a}^{\pi} \left(\frac{dY''}{dx}\right)^2 dx = \left| B^2 \left(\frac{x}{2} + \sin\frac{2x}{4}\right) + \frac{\beta^2 x}{a^2} - \frac{2\beta B \sin x}{a} \right|_{\pi - a}^{\pi}$$

Adding the losses for the curves Y' and Y'' it is seen that

$$\left.\frac{2\beta B \sin x}{a}\right|_{\pi}^{\pi - a} - \left.\frac{2\beta B \sin x}{a}\right|_o^a = 0$$

Thus, even here superimposition of losses is possible, since the term involving β and B concurrently goes out to zero for the superimposed fields. Thus, the entire flux distribution eddy current losses are superimposable. It follows furthermore that since any curve can be assumed as made up of an infinite number of straight lines it is only sufficient that the degrees of symmetry shown by Fig. 8 for the armature cross field to be present, for the eddy losses to be superimposable. The distortion caused by the variation in permeability along the pole-shoe periphery and in the pole-shoe will obviously upset the arrangement as to symmetry. This factor is however negligible. As regards the hysteresis loss a glance at the resultant curve of flux will show that the loss is not superimposable. Yet no great error will have been committed if a new value of 1.2 times the open circuit hysteresis loss is taken for the full load hysteresis loss. Turning now to obtaining a formula for the teeth iron eddy current incremental loss

$$\frac{10}{4\,\pi}\,.\,H\,.\,\delta = x\,\frac{\text{AB}}{\pi\,D}$$

In this equation H is the cross flux density in the air gap at any point, due to the ampere bars of the armature. Let

δ = width of gap in cm.

x = distance measured from midpole position. The maximum value of x will therefore be $\pi\,\frac{D}{p}$.

AB = ampere bars of the entire armature.

Now from the voltage equation, and the value of the current per circuit,

$$\frac{\text{AB}}{\pi\,D} = \frac{60 \times 10^8 \times W}{p\,R\,F} \times \frac{1}{\pi\,D}$$

Here W = watts output; D = diameter of armature; p = poles; R = revs, and F = flux per pole in *C.G.S.* lines. Thus, by above

$$H = \frac{4\,\pi}{10} \cdot \frac{60 \times 10^8 \times W}{\delta\,p\,R\,F \times \pi\,D} \cdot \frac{\pi\,D}{2\,p}$$

$$= \frac{2\,\pi}{10} \cdot \frac{60.\,10^8.\,W}{\delta\, p^2\, R\, F}$$

Since the pole-pitch factor is f and not $f = 1$ the actual maximum density will be

$$H_{\text{max.}} = \frac{2\,\pi\, f.\, 60.\, 10^8.\, W}{10\,\delta\, p^2\, R\, F}$$

A constant will be necessary for the straight line curve of distribution when referred to sine curve open circuit iron loss curves for the iron loss determination.

If $y = \sin x$

$$\int_0^{\frac{\pi}{2}} \cos^2 x\, d\,x = \frac{\pi}{4} \text{ and for } y = \frac{2\,x}{\pi}$$

so as to obtain the same maximum

$$\int_0^{\frac{\pi}{2}} \frac{4}{\pi^2}\, d\,x = \frac{2}{\pi}\,.$$

Thus the correction factor is $\frac{4}{\pi} \times \frac{2}{\pi} = \frac{8}{\pi^2}$. Again, since for the same frequency the variation is only over the range f instead of $f = 1$, an additional multiplier will be necessary and we obtain finally $\frac{8}{\pi^2 f}$. Thus considering only that part of the cross flux curve under the pole-shoe, the loss is, remembering that the loss in the teeth is due to $\frac{H}{r}$, where r is the tooth pitch ratio,

$$e\left(\frac{H}{r}\right)^2 \cdot \frac{R^2\ p^2}{120^2} \cdot \frac{8}{\pi^2 f} \cdot \text{volume of teeth}$$

Here e is the multiplier to reduce the losses to watts per cycle per seconds per cm^3. To this should now be added the losses in the interpolar region, or,

$$e\left(\frac{H}{r}\right)^2 \cdot \frac{R^2\ p^2}{120^2} \cdot \frac{8}{\pi^2 (1-f)} \cdot \text{volume of teeth.}$$

Adding these two losses together the total eddy loss =

$$e\,\frac{R^2\,p^2}{120^2} \cdot \frac{8}{\pi^2 f\,(1-f)} \cdot \left(\frac{H}{r}\right)^2 \cdot V$$

where V = volume of teeth.

Substituting for H its value, there is finally obtained

$$\text{teeth loss} = \frac{8\,e\,f}{(1-f)} \cdot 10^{14} \cdot \frac{W^2}{r^2\,\delta^2\,F^2\,p^2}$$

Professor Ernest Wilson (see *The Electrician*, February, 1904) has shown that the flux distribution loss in the core of an armature is independent of the flux distribution in the air gap. It follows therefore that the core need not be taken into account in determining the incremental eddy current loss. The value of e is

$$3.27 \times 10^{-13} \text{ thus}$$

Incremental eddy current loss

$$= \frac{262\ f\ [\text{volume of teeth}]}{(1-f)\ r^2\,\delta^2} \cdot \left(\frac{W}{p\,F}\right)^2,$$

but the specific teeth volume, *i.e.*, per $\pi\,D\,l\,i\,v$ is $\lambda\,r$, thus the final reduction is

Specific incremental loss

$$= \frac{222\ \lambda}{r^3\ \delta^2\ (D\ l)^2} \cdot \left(\frac{W \times 10^{-3}}{B/1000}\right)^2$$

CHAPTER VII.

THE CHARACTERISTIC CURVE AND STABILITY.

After a machine is designed it may well occur that at the working magnetic density a small change in the effective field pole ampere turns may cause quite a considerable change in the voltage at the brushes. The extent to which this may take place will depend upon the shape of the characteristic or, open circuit curve of volts or flux plotted against ampere turns on the poles. Where constant voltage is an essential, changes in voltage due to the back ampere turns of the armature when the brushes are not in the exact neutral position, can be overcome by altering the character of the open circuit curve by means of thin punched plates inserted between the pole and the yoke, or, between pole-shoe and pole. These plates, which may be called " stability plates," should of course be worked at a very high density with the normal working flux. In this manner, assuming the original no-load characteristic to have quite a convex character, the adoption of this device results in quite a flattening at the working portion of the curve. See Fig. 11 below. The curve *A* is the original open-circuit characteristic. The curve *B* is the transformed curve. The line *o a* represents the air gap line of ampere-turns against voltage (or flux). The line *o b* is the curve giving the ampere-turns additional to *o a* to be allowed for the stability plates. It is obvious that when holes are punched in these plates, for low densities the added reluctance is quite small, but as the induction is increased the very high densities obtaining in the stability plates (as much as $B = 30{,}000$) the plates are no better than an air layer introduced between the pole and the frame. So that when before, due to a given back ampere turns, the voltage would drop from $O\,D$ to $O\,E$, with the new curve B, and the same amount of back ampere turns, the drop is only to $O\,F$.

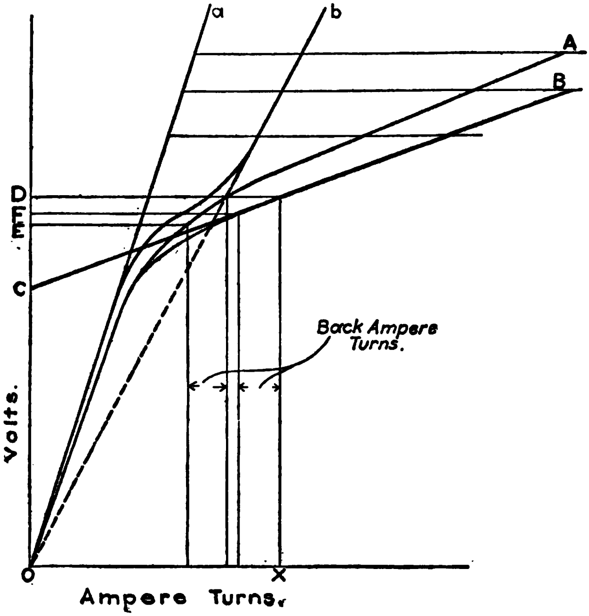

FIG. 11.—Effect of Stability Plates on Regulation.

In calculating for stability plates (they need not be more than three or four millimetres in thickness), it is advisable to calculate the flux in the iron for various values of **A** and to make graphically, an addition, to allow for the air cross section (for the same number of ampere turns). In this manner the complicated method of reluctances and the equivalent reluctance is avoided. Then, having added the ordinates of flux for the two curves for the same number of ampere turns, the resultant curve can easily be added to the original no load characteristic curve to give the modified curve.

A good rule determining the slope of the characteristic curve at the working density, can be made to depend upon the fractional variation of voltage, for a given fractional variation of ampere-turns. Let $\frac{dx}{ox}$ be the fractional variation of the ampere-turns and $\frac{dy}{o\,D}$ be the fractional variation of the voltage for the variation $\frac{dx}{ox}$. Here dx is an element of ox and dy is an element of $o\,D$. As a criterion for the stability of the machine, the ratio

$$\frac{\dfrac{dy}{o\,D}}{\dfrac{dx}{ox}}$$

may be taken as a measure. This really gives $\frac{dy}{dx} \cdot \frac{x}{y}$ since $o\,x = x$ and $o\,D = y$. Now $\frac{dy}{dx} = \frac{c\,D}{ox}$ and, therefore, the stability depends upon $\frac{c\,D}{ox} \cdot \frac{ox}{o\,D} = \frac{c\,D}{o\,D}$. This ratio for good design should be about ⅜. Obviously, the flatter the curve the less the voltage variation for the same variation of ampere-turns.* It is to be remarked that for a considerable range of the characteristic curve the ratio determining the stability above spoken of would be practically proportional to the distance of the point *x* from *o*.

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*See also paper by Messrs. Wimperis and Poynder on "Shunt Motor Voltage and Speed Relation in Engineering," Vol. lxxi., page 562.

CHAPTER VIII.

DESIGN OF COMMUTATORS.

Commutator design can be made to depend upon the condition of sparklessness, though the condition of minimum commutator losses will have some influence upon design. The greater the number of commutator segments, the better will be the commutating qualities, but there is a limiting condition of the minimum width of commutator bar possible. This lower limit is a 1 cm. width of commutator bar. In reality the diameter of the commutator will follow as soon as the voltage and character of the winding, whether lap or wave, is settled upon. Preferably, the maximum voltage should be taken, and then, since the voltage per commutator bar is known (this is gone into later), with a minimum width of commutator bar of 1 cm., the diameter results. If then, the most economical current density in the brushes is determined and the width of brush chosen, the length of the commutator will follow. The two important losses occurring on the commutator are brush friction loss, and, the electrical friction loss due to the contact resistance. Formulas for the determination of these two losses are the following:

Brush friction watts $= .000135\, D_c\, R\, \sigma$.
When $D_c =$ diameter of commutator,
$\sigma =$ area of all the brushes in cm.
$R =$ revs per minute.

Brush voltage drop in volts $= v = .149\,\Delta_b = \Delta_b\, f$, where $\Delta_b =$ current density per cm.^{2} of brush. It is thus seen that the total commutator losses in terms of σ can be represented by

$$\text{Total losses} = \frac{a}{\sigma} + b\,\sigma$$

Investigating for a minimum loss with $\frac{d}{d\,\sigma} = o$ we obtain

$$\frac{a}{\sigma} = b\,\sigma$$

Hence for maximum efficiency the brush current carrying loss should be equal to the brush friction loss.

If p = number of brush arms = number of poles and $2\,J$ = current per brush arm then,

$$.149 \times \frac{4\,p^2\,J^2}{\sigma} = .000135\,D_c\,R\,\sigma, \text{ and}$$

$$\Delta_b = \sqrt{\frac{D_c R}{1100}}$$

The actual current densities usually employed in practice vary from four to six amperes per cm.^{2} Thus, postulating a given voltage, the diameter of the commutator is derived by virtue of the reaction voltage and with the diameter known, the current density follows. A safe rule for the width of brush is about .09% of the pole pitch peripheral length of the commutator. Then, knowing the amperes, the net length of commutator follows from the width of brush. It should be the aim of the designer not to cover more than three segments of the commutator, and the minimum width of a carbon brush is roughly 1 cm.

CHAPTER IX.

THE SPARKING VOLTAGE S. V.

There are really two sparking constants: the one depending upon the voltage set up in the coil by virtue of the commutated current acting within the short-circuited coil whilst the brush is spanning two commutator segments; and the other, that due to the voltage maintained in the commutated coil, due to the limbs of the coil not residing in an absolutely neutral field. The first may be called the "sparking voltage," and since, usually, the voltage due to the commutated coil cutting lines of force hinders commutation, the latter may be called the field sparking voltage. (Designate respectively by S. V. and R. V.) In the subjoined figure let J = current per circuit;

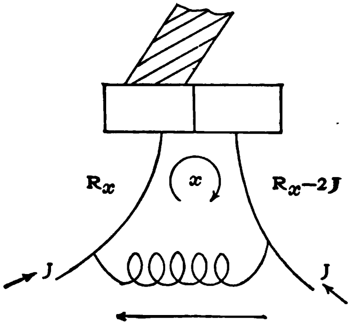

Fig. 12.—Commutated Coil Diagram.

x = circulating current due to commutation.
L = self-induction of short-circuited coil;
T = time of commutation;
dt = element of T;
e = voltage due to commutating field;
σ = area of brush.

Then, by Kirchoff's law, (See Fig. 12),

$$\frac{f\,x\,T}{\sigma\,(T-t)} + \frac{f\,(x-2\,J)}{\sigma\,t}\,T + L\frac{d\,x}{d\,t} = e$$

Since it is the sparking voltage at the instant the commutator leaves the brush that is the all important circumstance (glowing during commutation is not considered here), the above equation need only be investigated for time $t = T$ and when concurrently $x = 0$. In the limit, (and the above is the condition of perfect commutation),

$$-f\,\frac{d\,x}{d\,t}\,\frac{T}{\sigma} - \frac{2\,f\,J}{\sigma} + L\,\frac{d\,x}{d\,t} = e \text{ and}$$

$$L\frac{d\,x}{d\,t} = S.\,V. = \frac{v+e}{1-\dfrac{v\,T}{2\,J\,L}} < 10 = \frac{v+e}{1-\dfrac{\cdot 15\,T}{\sigma\,L}}$$

The quantity 10 is derived from actual practice. It will be seen that since $\frac{v}{J}$ = constant, for a given value of σ (by the expression for the contact voltage), increasing the current density, increases the sparking voltage. The value of v is, that derived from considering the current density employed in the brush with total current per brush arm $2\,J$. In the above the specific resistance of the brush is assumed to remain constant. Had the contact voltage been assumed independent of the current density, the equation of voltages by Kirchoff's law would result in $L \frac{d x}{d t} =$ constant. Thus a straight line law results for the variation of the commutation current. Further it has been usual to employ the limiting condition set by $1 - \frac{v T}{2 J L} = 1 - \frac{.15 T}{\sigma L} > o$. It is seen that here the current density in no wise enters into this condition of finite reactance voltage. Yet, it is known that the current density employed exerts a strong influence upon the commutation, and, hence the form given above.

CHAPTER X.

THE FIELD SPARKING VOLTAGE R. V.

Due to the field set up by the remaining armature conductors carrying current the short circuited commutated coil finds itself in a position where it is cutting lines of force. It is the magnitude of the cross flux voltage thus set up that has been designated by *e*. The magnetizing force of the armature ampere turn will

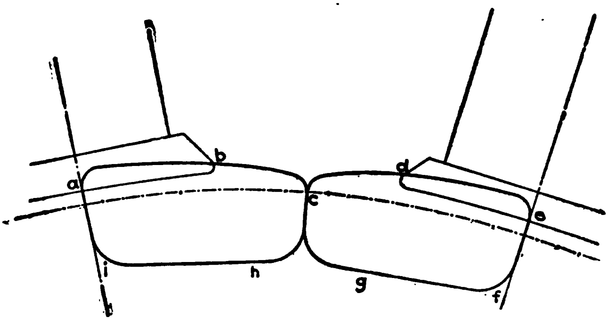

FIG. 13.—Integration Curves for Armature Cross Field.

cause the greatest force to appear about the position where commutation takes place. Due, however, to the interpolar space being void of iron, the flux that will be actually present will be relatively small. It will however still have an important bearing upon the state of commutation. Assume the conductor to be positioned etc (Fig. 13.) It is obvious that the entire path *a b c d e f g h*, is equivalent to two smaller circuits *a b c h i* and *c d e f g*. The real path of a magnetic line would be from one interpole position to the next interpole position through the pole-shoe, and, returning through the armature core. Of the two smaller circuits, the iron reluctance is negligible compared to the reluctance of the air path, and thus, for the magnetomotive force due to the conductors between *c* and *f*, the only part of the path that need be taken into account is *c d*. Correspondingly it will be *c b* for the conductors lying between *c* and *i*. The point *c* assumes the conductor lying beneath the brush.

Representing the space *c f* by *x* where the points *f* and *i* are practically at midpole position, the distance

$i f = \tau = \frac{\pi D}{p}$ and $i c = \tau - x$. Thus,

$$\frac{4\pi}{10} x \times \frac{\mathrm{AB}}{\pi D} = \int_c^d H_x l, \text{ also,}$$

$$\frac{4\pi}{10} (\tau - x) \frac{\mathrm{AB}}{\pi D} = \int_c^b H_{(\tau-x)} l.$$

Solving for H_x and $H\ (\tau - x)$, equal to B_x and $B_{(\tau-x)}$ respectively,

$$B = B_x + B_{(\tau-x)} = \frac{4\pi}{10} \cdot \frac{\mathrm{AB}}{\pi D} \cdot \left(\frac{\tau}{2bc} + \frac{\tau}{2bd}\right).$$

For the circumstance that $b\,c = b\,d$, since $b\,c + b\,d = \tau\,(1+f)$,

$$B = \frac{4\pi}{10} \cdot \frac{\text{AB}}{\pi D} \cdot \frac{2}{(1-f)} = \frac{8}{10} \cdot \frac{\text{AB}}{D\,(1-f)}$$

If ν = number of commutator parts.

$$R.\,V. = \frac{\text{Bars}}{\nu} \cdot \frac{\pi D R}{60} \cdot \frac{8}{10} \cdot \frac{\text{AB}}{(1-f)\,D} \cdot liv\,10^{-8}$$

Considering lap wound machines,

$$\text{armature bars} = \frac{E \times 60 \times 10^8}{R\,F}$$

The current per circuit is $\frac{W}{p\,E}$, hence,

$$\text{B} = \frac{W \times 60 \times 10^8}{p\,R\,F}$$

Hence

$$e_l = \frac{E \times 60 \times 10^8}{R\,\nu} \times \frac{\pi D R}{60} \times \frac{8}{10} \times \frac{W \times 60 \times 10^8 \times l\,i\,v \times 10^{-8}}{p\,R(1-f)D\left\{\frac{\pi D f}{p}\,livrB_a\right\}^2}$$

$$= \frac{E \times 48 \times 10^8\,p\,S}{\nu\,r^2 \pi f^2 (1-f)\,iv\,(B_a)^2}$$

$$= \frac{E}{\nu} \cdot p \cdot \frac{48}{100\,\pi\,f^2\,(1-f)\,i\,v\,m^2\,r^2} \cdot \frac{S \times 10^2}{(B/10.000)^2}$$

$$= 1.31 \cdot \frac{E}{\nu} \cdot p \times \frac{S \times 10^2}{r^2 (B/10.000)^2}$$

with $m = 1.05\,f = .7$ and $i\,v = .72$

It should be noticed that the value of e will be a minimum for $f = .666$ other things being equal. Since

$$e \equiv \frac{1}{f^2\,(1-f)}$$

taking $\frac{d}{d\,f} = 0, \quad \frac{2}{f^3\,(1-f)} = \frac{1}{f^2\,(1-f)^2}.$

Thus $f = \frac{2}{3} = .666.$

Considering wave wound machines

$$\text{Armature bars} = \frac{E \times 60 \times 10^8}{R\,F} \times \frac{2}{p}.$$

$$\text{Current per circuit} = \frac{W}{p\,E}.$$

$$\text{AB} = \frac{W}{p\,E} \times \frac{E \times 60 \times 10^8 \times 2}{R\,p\,F} = \frac{120.\,10^8.\,W}{p^2\,R\,F}.$$

Hence for wave wound machines

$$e_w = 2.62\,\frac{E}{\nu} \cdot \frac{S \times 10^2}{r^2\,(B/10.000)^2}$$

or more exactly

$$e_w = \frac{E}{\nu} \cdot \frac{96}{100\,\pi\,f^2\,(1-f)\;i\,v\,m^2} \cdot \frac{S \times 10^2}{r^2\,(B/10.000)^2}$$

It may be convenient to bring the two cases under one heading by taking the variable J, the current per circuit, into account. Let t = turns per coil, or turns per commutator bar

$$R.V. = \frac{32 \times \pi\,i\,v\,J\,t^2\,l\,R\,10^{-8}\,\nu}{600\,(1-f)} < 6$$

The limit of six volts is that found by experiment. By the former evaluations it will be seen that the R. V. depends upon the voltage of the machine per bar of the commutator, the design coefficient S and the density employed in the teeth of the armature. If in the equation for S. V. a given current density will be employed (with a current density of six amperes per cm.^{2} the value of v is about unity) the limiting condition expressed by $1 - \frac{v\,T}{2\,J\,L}$ will permit of expressing a simple relation between the R. V. and this sparking condition (designate the latter by S. C.). Taking $v = 1$

$$1 - \frac{T}{2\,J\,L} > 0, \text{ hence } \frac{T}{J\,L} = S.\ C. < 2.$$

Let inductance per turn $= L$ and let $t =$ turns per coil, also $\nu =$ number of commutator bars.

$L\,t^2 =$ inductance per coil.

$$T = \frac{1}{\frac{R}{60}}\,\nu$$

Hence condition $S.\ C. = L.\ t.^2\ J.\ \frac{R}{60}\ .\ \nu < \frac{1}{2}.$

From experiment it is learned that

$$L.\ t.^2\ J.\ \frac{R}{60}\ .\nu = S\,C < 2$$

is the limit of sparkless commutation.
Thus

$$\frac{R.\ V.}{S.\ C.} = \frac{32\,\pi.\ i.\ v.\ l.\ t.^2\ J.\ R.\ \nu\ 10^{-8} \times 60}{600\ (1 - f)\ L\,t^2\,J\,R\,\nu}$$

Assuming a (about 4) lines per ampere per cm. length of imbedded conductor and one line per cm. for the end connection length, since the length l has to be multiplied by $\left(1+\frac{2\pi\varepsilon}{\sqrt{3}}\right)$ to obtain the length of an effective bar, it follows, that

$$L = \left\{2\,a\,l+\frac{2\,l}{4}\cdot\frac{2\,\pi\,\varepsilon}{\sqrt{3}}\right\}10^{-8}$$

$$= l\left\{2\,a+\frac{\pi\,\varepsilon}{\sqrt{3}}\right\}10^{-8}\cdot$$

$$= M\,l\,10^{-8}$$

Thus

$$\frac{R\ V}{S.\,C.}=\frac{3\,2\,\pi\,i\,v.}{10\,(1-f)\,M}$$

For $a = 4$ and $\varepsilon = .5$, $(1-f) = .3$ and $i.v. = .72$

$$\frac{R.\,V.}{S.\,C.} = 2.69.$$

Thus working with a given current density and using S. C., it is seen that one criterion is as good as the other.

CHAPTER XI.

ARMATURE SELF INDUCTION.

In determining the armature self induction the following formula can be employed:

$$\left.\begin{array}{l}\text{Self induction per ampere turn per}\\ \text{cm. length of imbedded conductor}\end{array}\right\} = \frac{.38\,(\rho+6)}{\sqrt{q}}$$

where ρ is the ratio of slot depth to slot width for open type slots and q is the number of slots per coil side. For the end connections the number of lines per centimeter of coil length per ampere turn can be taken as one line. In reality the self induction per turn is very much influenced by the flux density in the armature. Even in the interpole position, which is the case where the coil undergoes commutation, a strong field will considerably diminish the self induction. This is shown by the three curves. (See Fig. 14.) They represent respectively the self induction of a commutated coil at the neutral (interpolar) position of the brushes, and for field strengths of zero, normal, and a field due to twice the normal number of ampere turns. The influence of the magnetic density is easily seen.

The curve (Fig. 15) shows the influence of lag or lead of the brushes on the self induction. Normal field strength and normal armature current were employed.

It is seen that for 10% variation of the brushes from the neutral position either way the self induction can be assumed to be constant. Beyond this point the self induction rapidly increases, attaining as much as 100% increase. For good commutation a strong field is essential though after having obtained a fair degree

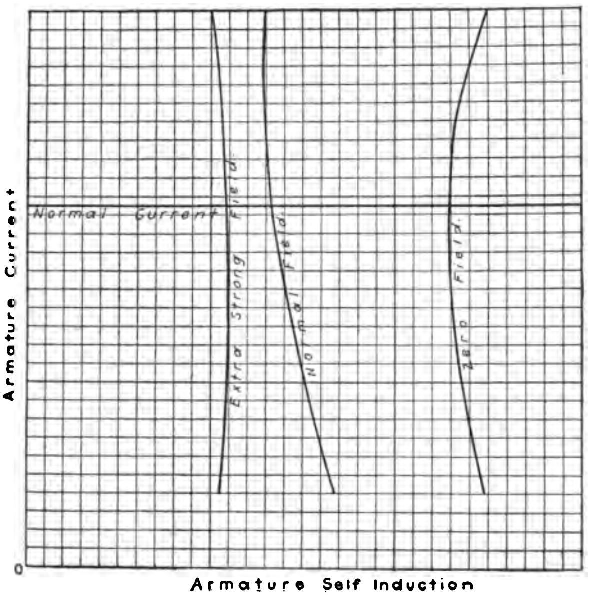

FIG. 14.

of saturation increasing the field ampere turns no longer diminishes the self induction in as rapid a proportion as would repay for the increased excitation loss.

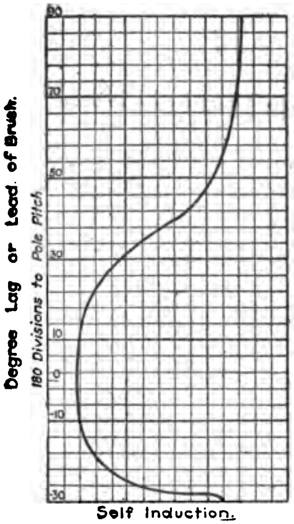

FIG. 15.—Commutated Coil Self-Induction Curves.

CHAPTER XII.

THE SLOT AND WINDING DIAGRAM.

It is oftentimes necessary to construct a winding table for quite long ranges of speeds and voltages. Since the density in the teeth only varies with the "design coefficient," the pole-pitch, and the air gap length, it will mean that for a given design the flux, and, hence the density, will be kept constant. Engine speeds for a given output are usually quite definite, but belted speeds permit of great latitude. Thus in the matter of voltage the range can be taken of from 80 to 550 volts as consistent with possible commercial requirements. Having settled upon the tooth pitch ratio and the depth of slot that will meet with the most important requirement of a given carcase, it will behove in the matter of core stampings, to choose such numbers of slots as will be consistent with the multiple character of the windings, and thus require as small an assortment of stampings as is possible. In keeping the tooth pitch ratio and the depth of slot constant the relative value of iron and copper losses is held intact. This is a worthy consideration from the point of view of the foregoing investigations. It is obvious that with a variety of sets of armature windings such numbers of slots should be chosen as will divide evenly the necessary sets of armature coils.

In stepping from one set of conductors to another, gaps no larger than 10% will be sought, and, it will thus mean, that the field flux would in the worst cases not have to be varied more than this percentage. The number of coils per slot varies usually from two to four, but the practicable range is from one to six. Again, the number of slots may be odd or even. The advantage of an odd number of slots is to avoid the "pumping" action due to a number of slots divided evenly by the number of poles. If an odd number of slots is chosen it should be nearest that number that will result in as few dummy coils as possible. The number of slots under each pole should not be less than seven or eight. The minimum width of slot may be taken as nine millimetres, although the real limit is set by the increasingly bad copper factor and high inductance of very narrow slots. The maximum number of commutator bars, and, hence of the number of armature coils is fixed by the minimum width of commutator bar, which is one centimetre. The determination of the winding table spoken of is no easy task where a trial and error method is adopted, and it was with a view of facilitating this arduous task that the following graphical method was evolved. Adopting the example method of elucidation let the following conditions be paramount:

Voltages
125
250 at 375 revs. } (low speed.)
500

Voltages
125
250 at 700 revs. } (high speed.)
500

Obviously the number of conductors necessary in the above assuming a normal flux of 4.6 mega lines are thusly

$$\left.\begin{array}{l} 435 \text{ (lap wound)} \\ 870 \text{ (lap wound)} \\ 1740 \text{ (lap) or } 870 \text{ (wave wound)} \end{array}\right\} \text{(low speed)}$$

$$\left.\begin{array}{l} 216 \text{ (lap wound)} \\ 435 \text{ (lap wound)} \\ 870 \text{ (lap) or } 435 \text{ (wave)} \end{array}\right\} \text{(high speed)}$$

Proceeding with the best condition, *i.e.*, one turn coils, this would necessitate $\frac{216}{2} = 108$ coils. (Thus, sets of these coils might be used.) Observing the rules indicated above, it will be assumed that it was found unsuitable to adopt this number of slots since it necessitated a slot width of less than nine millimeters. Thus, trying now two one turn coils per slot, (instead of as before one, one turn coil) the number of slots is diminished to 54 (take 55 since it is odd). This latter is found to suit all requirements. Fifty-five is found to cover the cases above enumerated, being very nearly divisible into all the above sets of conductors. In the case of 435 conductors a 14 coil (lap wound) could be used and for the case of 1740 conductors a four-turn coil (wave) would be found all sufficient. It is obvious, the fewer the number of turns per coil, the cheaper the construction; and the fewer the stock of coils necessary to be held on hand. Consider now the extreme numbers of sets of conductors necessary. Take firstly the maximum voltage at the minimum revolutions and then again the minimum voltage at the maximum number of revolutions; 550 volts at 350 revs.

requires 1850 conductors and 80 volts at 1000 revs. demands 104 conductors. It will easily be seen this just fits the condition of a 55 slot stamping. The start in the method will then be made with 55 slots. Mark off a length on cross section paper on a slot scale of 55 to represent 55, and along this same line on a new scale mark off by two's a *conductor-per-slot scale*, starting from two conductors per slot (which is the minimum possible) up to any convenient number dependent upon the maximum number of conductors. (See Fig. 16.) The conductor per slot scale is again marked off into turn regions by vertical lines (see Fig. 17). The position of the vertical lines is determined by the number of commutator parts possible.

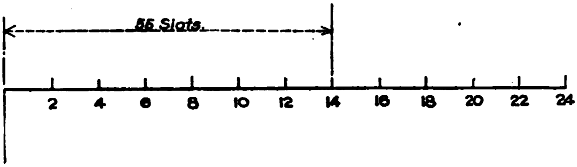

FIG. 16.—Slot and Winding Diagram.

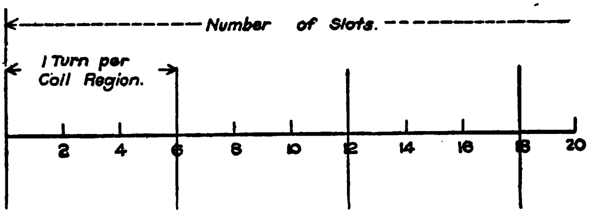

FIG. 17.—Slot and Winding Diagram.

The number of turns possible per coil depends upon the maximum number of coils that can be connected to the commutator. However, the number of commutator parts is limited, hence the number of armature coils is limited and since a given number of slots has been taken, the maximum number of conductors per slot is determined. The maximum number determines the position of the vertical lines and the maximum number of coils is of course given by

$$\frac{\text{Diameter of commutator}}{\text{Minimum width of commutator bar}} = \left\{ \begin{array}{l} \text{Maximum} \\ \text{number of} \\ \text{armature coils.} \end{array} \right.$$

An example will make this more evident.

Due to the diameter of the commutator let the maximum number of coils be say, 157. If single turn coils only are to be considered, it will mean that only $157 \times 2 = 314$ conductors will be possible, and since there are 55 slots, this will mean that no more than $\frac{314}{55} = 6$ conductors per slot will be possible. Hence the vertical line marking off the single turn region must be drawn at 6. If now two turns per coil is postulated the maximum number of conductors will be increased to $2 \times (157 \times 2) = 628$. (The maximum number of coils is of course always being fixed at 157.) Since as before the number of slots is 55 it means that the maximum number of conductors per slot is $\frac{628}{55} = 12$. Now here is where one of the advantages of the graphical method outlined is evidenced. Considering two turns per coil it is obvious that the possible number of conductors with a 55 slot stamping must be divisible by four. Hence although an eight-conductor per slot combination is possible, a ten conductor per slot is impossible since it is not divisible by 4. Thus along the conductors-per-slot line in the two turn region only 8 and 12 are marked off. The intermediate number 10 is not shown. In the three-turn region the maximum number of conductors is $3\times2\times157=942$, with a maximum of $\frac{942}{55}=17.2$; or, take 18 conductors per slot. The vertical line is thus drawn at 18. They will likewise be drawn at 24, 30, etc. Between 12 and 18 in the three-turns-per-coil region there are no even numbers divisible by $3\times2=6$ and hence only 18 at the end of the three-turn region is shown on the conductor per slot scale drawn with even dimensions. In the four-turn region it will be found that between 18 and 24 there is only 24 that is divisible by $4\times2=8$. In the five-turn region, between 24 and 30, curiously enough only 30 again makes the combination that is possible for a five-turn coil with ten conductor sides. The number 20 which is divisible by 10 occurs in another region and hence can not be shown. The point 30 is then the only number marked. In the six-turn region which will be the last region considered, since no coils with greater number than six turns are made, the number 36 marks the ends of the six-turn region, and, also marks the only number between 30 and 36 that is divisible by $12=6\times2$. Having marked off the verticals, indicating the turn-regions, and also the sets of conductors per slot for the given slot number indicated by the slot line magnitude drawn on the continuation of the conductors-per-slot line, the next step is to take some point as origin, preferably some point along the first perpendicular marking zero conductors-per-slot, and then, draw radial lines (conductors-per-slot lines) from this origin passing through the respective conductors-per-slot marked off (or indicated) on the 55 conductors-per-slot line. See Fig. 18.

It will be obvious that the radial conductors-per-slot lines need only be continued into the six-turns-per-coil region. Further the radial lines are shown dotted in those regions where such a number of conductors per slot with the turns per coil indicated are impossible.

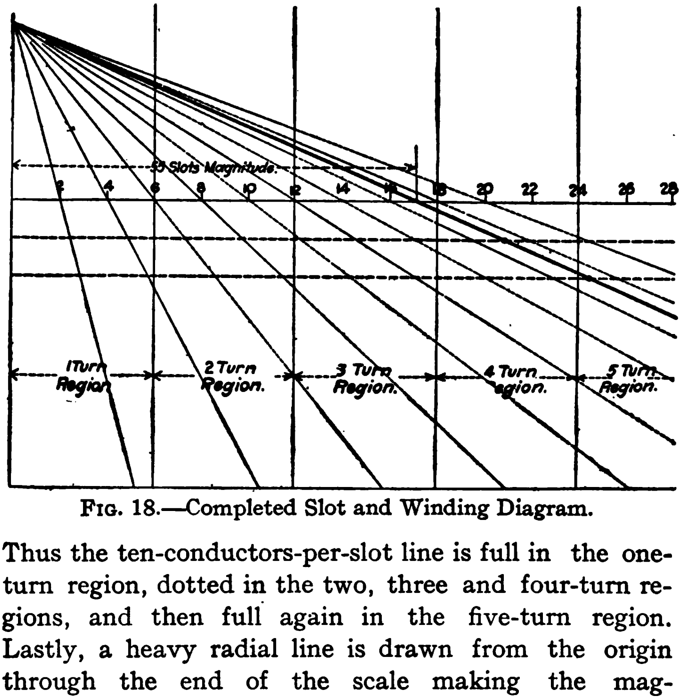

FIG. 18.—Completed Slot and Winding Diagram.

Thus the ten-conductors-per-slot line is full in the one-turn region, dotted in the two, three and four-turn regions, and then full again in the five-turn region. Lastly, a heavy radial line is drawn from the origin through the end of the scale making the magnitude of a 55 slot stamping. Now any horizontal line drawn from the zero vertical line to this slot-per-stamping radial line will indicate a new number of slots per stamping, and will be measured by its magnitude, relative to the length of the 55-slot-per-stamping horizontal line. It will now be obvious that such new slots-per-stamping lines should be drawn as will cross as many of the conductors-per-slot radial lines as possible. Such horizontal lines can then be measured off, and the number of the slots-per-stamping determined. In fact this graphic method indicates the futility or even possibility of a generous winding table. Suggested new numbers of slots per stamping are shown by the dotted horizontal lines. The whole is a graphic view of the winding possibilities of the armature. Where the suggested dotted horizontal lines cross the radial lines new combinations for the winding table with the new number of slots are indicated. To see how evenly the stepping is from one set of conductors to the next possible, the new points should be projected vertically down or up to the base line (the first conductor-per-slot line, in this case the 55-slot conductor-per-slot scale) and such projected points will be equivalent on the scale to which the points are projected, to such numbers (usually odd or fractional numbers) of conductors per slot with the number of slots (55 in this case) indicated by the line. The intervals from one set of conductors to the next set (in all probability a projected point) should not be greater than a 10% interval. Thus starting from four conductors-per-slot the next point should be 4.4 conductors-per-slot, or say 5. Evidently this point will have to be a projected point due to a suggested horizontal line cutting a radial line other than the five-conductors-per-slot radial line. Usually not more than three sets of stampings are necessary, but the winding and slot diagram should be held for reference, and would thus take the place of a winding table.

CHAPTER XIII.

METHOD OF DETERMINING THE LAW OF IRON LOSS AND FRICTION.

For determining the law of iron loss and friction four observations are necessary. With the field excitation constant, observe the watts required to run the machine on a given load and at a given speed. Then, with same excitation take some radically different speed and observe the watts necessary to overcome the losses whose law it is wished to determine. The copper losses are of course to be eliminated.

In the following it is assumed that the watts necessary to overcome the frictional losses vary as the speed of rotation to some constant index. Thus let $w =$ watts at any speed ω, then,

$$w = a\,\omega^{x}$$

where a and x are constants.

Making the first two observations let these be W and Ω respectively. Then

$$W = a\,\Omega^{x} \text{ and } a = W\,\Omega^{-x}$$

$$\text{Now torque} = \frac{\text{Watts}}{\omega} = a\,\omega^{x-1} = -I\,\frac{d\,\omega}{d\,t},$$

where $I =$ moment of inertia. From the equation for torque, where $a\,\omega^{x-1} = -I\,\frac{d\,\omega}{d\,t}$, it follows, that

$$\int_{0}^{\Omega} \omega^{1-x}\, d\,\omega = -\frac{a}{I}\int_{T}^{0} d\,t$$

where T is the time in seconds that the armature takes to come to rest, (or to a lower limit if necessary), when the current from the armature is cut off and the frictional losses due to excitation and the bearing friction are still allowed to operate. Thus, from the integration

$$\frac{\Omega^{2-x}}{2-x} = \frac{a\,T}{I} \text{ and therefore}$$

$$\frac{w}{W} = \left(\frac{\omega}{\Omega}\right)^{\left(2-\frac{I\,\Omega^2}{W\,T}\right)}$$

Taking the logarithm of both sides of this equation

$$\log w - \log W = \left(2 - \frac{I\,\Omega^2}{W\,T}\right)(\log \omega - \log \Omega)$$

$$\text{and } \frac{I\,\Omega^2}{W\,T} = 2 - \frac{\log W - \log w}{\log \Omega - \log \omega}$$

Thus taking two additional observations w_1 and ω_1 the terms I and T can be eliminated and one thus has finally

$$\frac{w}{W} = \left(\frac{\omega}{\Omega}\right)^{\left(\frac{\log W - \log w_1}{\log \Omega - \log \omega_1}\right)}$$

Here w is the watts necessary for any speed ω. It is obvious that with the formula a determination can be made of the law of friction loss above and thus of the iron loss not involving the other frictional loss. In this latter a calibrated motor for efficiency is necessary. The calibrated motor should be preferably just sufficient to overcome the losses that are to be measured in the larger machine under observation. Running the second machine at the higher speed first the watts and speed are observed and then the same is done at some lower speed. The excitation is first allowed to operate with the bearing friction and then later the observations are made without excitation.

CHAPTER XIV.

THE HEATING TIME CONSTANTS.

Any machine starting up from cold in which frictional losses are taking place must gradually rise to a temperature above that of the surrounding medium. Theoretically, the law of cooling follows from the Newtonian law that the rate of change of temperature is proportional to the difference between the temperature of the body and that of the surrounding conducting air. If t is the time and τ is the temperature rise

$$\frac{d\tau}{dt} = \frac{\tau_{max.} - \tau}{T}$$

In this equation τ_{max} is the maximum or final temperature rise to which the body will approach. The quantity T is a constant and is called the heating time constant. It is measured in minutes (or hours). It expresses the time that the body would take to reach the final temperature rise if all the heat generated were conserved in the body, *i.e.*, without any dissipation. Solving the above differential equation there results

$$t = \tau_{max}\left(1 - e^{-\frac{t}{T}}\right)$$

This is the equation of a logarithm curve. It will be observed the subtangent for any point of the curve is a constant. It is no other than T, the time constant. Since its magnitude can be obtained by drawing a tangent at the origin the definition of time constant readily follows.

The line $o\,A$ (Fig. 19) represents the straight line law of rise of temperature if there were no conducting away of heat. It is obvious that the curve of cooling will be the exact inverse of the heating curve. Furthermore, for intermittent running a knowledge of the two curves and the time constant of the machine is absolutely necessary in order to calculate the final rise of temperature where a particular time load factor is given.

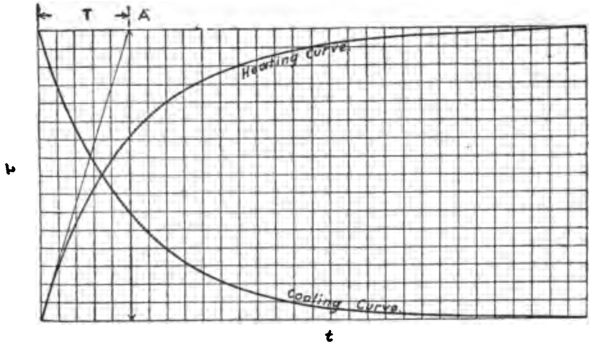

FIG. 19.—Logarithmic Heating and Cooling Curves.

Plotting the logarithmic curve against the percentage of the final temperature rise, and $\frac{t}{T}$, any machine started up from cold with a time constant of T minutes will reach the corresponding percentage of the final temperature rise in t minutes if the percentage is read off corresponding to the point $\frac{t}{T}$ of the curve.

It will be of value to evolve formulas for the time constants of open type armatures and the corresponding field coils. Taking the circumstance when the entire heat generated is conserved within the body it is obvious that the final temperature rise τ_{max} will be given by the following:

$$\frac{T}{\tau_{max.}} = \frac{\text{Heat capacity of body}}{\text{Heat generation of body}}$$

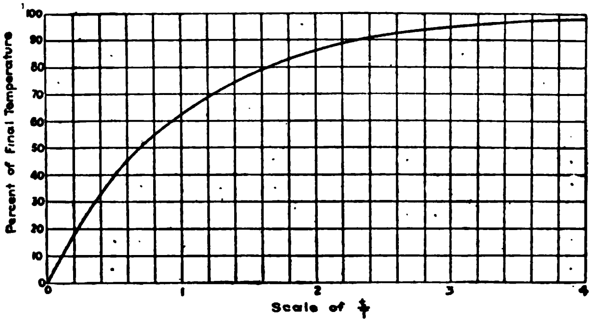

FIG. 20.—Temperature Rise Curve.

Since the heat capacity is proportional to the volume, and the volume can be taken as roughly proportional to $D^2\,l$; further, knowing that the permissible generation of heat depends upon $\pi\,D\,l$ and the peripheral speed, since

$$\frac{\text{Watts}}{\text{Surface}} \equiv D.\,\text{revs} + b. \quad \text{(see Fig. 1).}$$

$$T \equiv \frac{D^2\,l}{D\,l\,(D\,R + b)} = \frac{\text{Constant}}{R + \dfrac{b}{D}}$$

The constant b has been employed in order that the expression may serve for both a low and high speed machine. From experimental data on open type machines the formula for T in minutes is

$$T = \frac{36000}{R + \frac{2000}{D}}$$

If the speed R is taken to be zero $T = 18\,D$. On enclosed machines this latter condition can always be said to approximate. The exact value of the time constant will depend upon the particular type of machine. The usual range is given by $T = (5 - 10) \times D$. For open type field coils the following formula can be employed:

$$T = \frac{72}{\left(\frac{\Delta}{100}\right)^2}$$

where Δ is the current density (gross) in the field copper per square centimeter. The justification of the above formula is as follows:

$$T \equiv \frac{\text{Volume}}{\text{Watts generated.}}$$

If Λ = length of mean turn of the field coil and A = the (Guldinus) cross section of the field coil, the volume of the coil is given by $A\,\Lambda$. On the other hand, the watts generated are proportional to $\mathbf{A}^2 \times \frac{\Lambda}{A} = \Delta^2 A\,\Lambda$. Hence

$$\frac{\text{Volume}}{\text{Watts}} \equiv T \equiv \frac{A\,\Lambda}{\Delta^2 A\,\Lambda} = \frac{1}{\Delta^2}$$

But Δ is the current density (gross) in the field coil; hence the above.

CHAPTER XV.

TEMPERATURE RISE DETERMINATION OF ENTIRELY ENCLOSED MOTORS.

As has been already pointed out, the temperature rise of open type armatures has been found to depend upon the watts per square unit of cylindrical surface generated, and upon the peripheral speed. In the open type machines the air is expelled from the armature into the outside air and thus serves as a carrier for the heat of the armature body. In the enclosed type the same volume of air is stirred up into a vortex motion, and the heat of the armature body, that of the field coils, and even that of the commutator is imparted to the same body of air in cyclic order. The great difficulty with enclosed motor temperature rise determination has been the neglect of a study of the air as a carrier of the internal heat to the motor casing. It is obvious that when a given stirring up of the enclosed air is reached the heated surface of armature field and commutator may be said to be in almost perfect communication with each other. Thus the temperature rise of the individual parts will be dependent upon the peripheral speed of the armature in transmitting the different heats to the motor casing, and then, when the critical peripheral speed of the motor is reached the temperature will become uniform. The excess heat in any one part will be evenly distributed to all the surfaces till the proper uniform temperature rise is reached consistent with having passed above the critical speed at which the air is almost perfectly stirred. The following formulæ can be said to hold for enclosed motors.

Armature temperature rise C° (barrel surface alone considered).

$$= 34{,}500\,\frac{\dfrac{\text{watts}}{\text{cm}^2}}{p.\,s. + 1780}$$

The watts to be taken for the armature barrel surface would be the armature copper loss and one-half the iron loss watts. The remainder is radiated otherwise than through the barrel surface.

Temperature rise in C° for enclosed motor field coils where the all round radiating surface is employed.

$$\frac{= 217{,}000 \times \dfrac{\text{watts}}{\text{cm}^2}}{p.\,s. + 4400}$$

The constants of this formula were obtained for watts per square centimetre equal to .065.

Temperature rise in C° for commutator of enclosed motors (barrel surface alone considered).

$$= \frac{4.650 \times \dfrac{\text{watts}}{\text{cm}^2}}{p.\,s. + 650}$$

The $p.\,s.$ = peripheral speed in cm. per second.

If the peripheral speed at which the armature is to run is much below 900 cm. per second, the above formulæ will determine the temperature rises for the individual parts. If, however, the critical speed of 900 cm. per second is reached or even passed the field watt losses, the commutator losses and as much of the armature losses should be taken as will give the same temperature rise for the particular peripheral speed at which the armature is run. Then the remainder of the watts generated is to be evenly distributed among the different surfaces, and, in the proportion of the watts per square unit of surface at which a uniform temperature was obtained. The proportionate increase in the watts per unit of surface will give the final temperature rise throughout the different parts. Should, however, the field watts be diminished for the higher speeds, allowance will have to be made for the increased conductivity of the coil. Obviously, the temperature would then be lower than the armature or commutator.

CHAPTER XVI,

(In the following a certain amount of repetition will be found.)

FLUX PER POLE.

Consider the teeth under the pole-shoe. A certain amount of flux will directly cross the air gap to the tops of the teeth but a certain amount can be assumed to pass through the air of the slots (parallel to the sides of the teeth), to the bottom of the slots. It is an approximate method, but for the exigencies of design is sufficiently exact. The method is due to Messrs. Parshall and Hobart. Thus

Total flux = teeth flux + slot air flux.

Let B = density in teeth, and also, let

$$\text{Apparent density in teeth} = B_a = \frac{\text{Total flux}}{\text{Teeth cross section}}.$$

Hence

$$\frac{\text{Total flux}}{\text{Teeth cross section}} = B_a = \frac{\text{Teeth flux}}{\text{Teeth cross section}} +$$

$$\frac{\text{Slot air flux}}{\text{Slot air cross section}} \times \frac{\text{Slot air cross section}}{\text{teeth cross section}},$$

$$\text{and } B_a = B_r + H_r\left(\frac{1 - vir}{vir}\right), \text{ if}$$

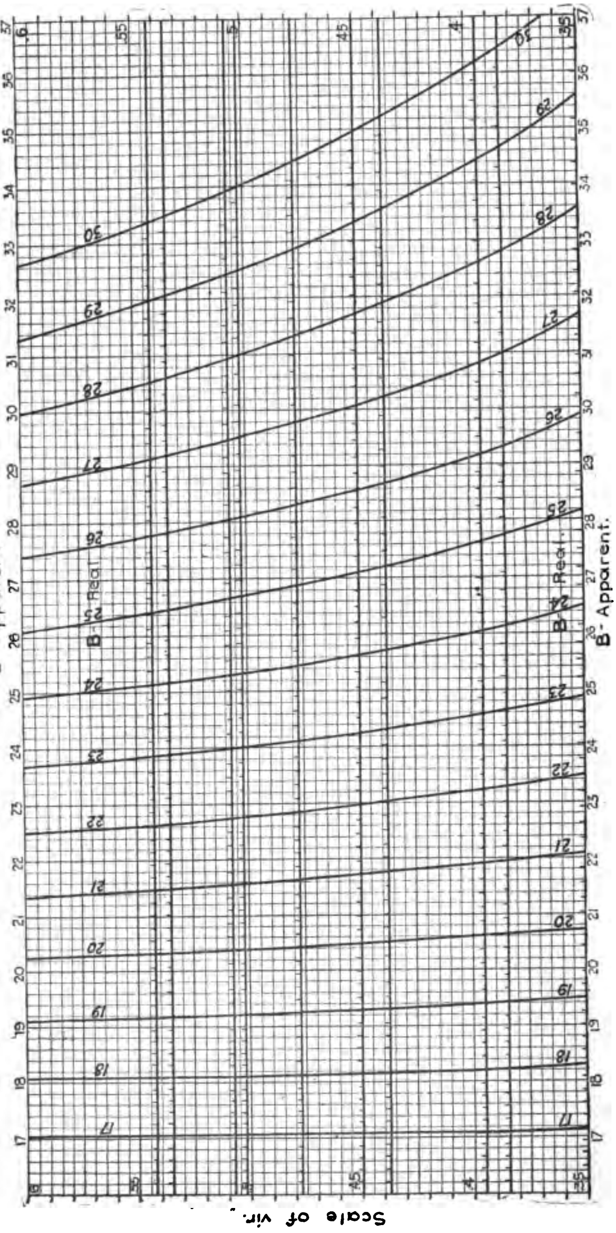

FIG. 21.—Tooth Density Curves.

i = iron factor = about .9 (due to scale, distance between plates, etc.).

v = ventilating factor rising to .8 in value for 20% ventilation.

r = ratio of tooth width, to tooth + slot width.

B_r = real density in teeth and H_r density in air to correspond to B_r.

$$\text{Flux per pole} = \frac{\pi D l r i v f}{p} B_a$$

$$= \frac{f \pi D r}{p} l i v \left\{ B_r + H_r \left(\frac{1 - vir}{vir} \right) \right\}$$

EMPIRICAL FORMULA FOR TOOTH PITCH.

$$\textit{Tooth pitch} = z = \frac{\pi D}{n} = 2 + \frac{D}{100} \text{ (cm.)}$$

EMPIRICAL FORMULA FOR DEPTH OF SLOT.

$$\textit{Depth of slot} = \lambda = 2 \log_{10} D \text{ (cm.)}$$

FIELD COIL CURRENT DENSITY.

The maximum permissible current density for field windings with a temperature rise of 40° C is in reality dependent upon the radiating surfaces of the coils. It is, however, more convenient to use a formula involving current density per cm.^{2} and a given winding depth than one for which the radiating surface has to be calculated for each case. Let Δ = gross current density in the field coil and let A = cross section of coil, also let Λ = length of mean turn of coil. The watts lost per coil are thus

$$\text{Field watts} = a\,(\Delta\,A)^2 \times \frac{\Lambda}{A}$$

$$= a\,\Delta^2\,A\,\Lambda$$

but $A\,\Lambda$ = volume of coil $= \frac{\pi}{4}\,(D^2 - d^2)\,h$ when D and d are the external and internal diameters of the coil and h is the height of the coil axially.

Cylindrical surface $= \pi\,D\,h$

$$\text{Hence } \frac{\text{watts}}{\text{surface}} = \frac{a\frac{\pi}{4}\,(D+d)\,(D-d)\,h\,\Delta^2}{\pi\,D\,h},$$

and since $D+d$ is practically equal to $2\,D$, which will be the case in ordinary dynamo practice,

$$\frac{\text{watts}}{\text{surface}} = a\,\left(\frac{D-d}{2}\right)\,\Delta^2 \text{ (very nearly).}$$

But $\left(\frac{D-d}{2}\right)$ is nothing other than the winding depth, and since $\frac{\text{watts}}{\text{area}}$ = constant, for a given temperature rise and normal dynamo constructions,

$$\Delta = \frac{\text{amperes}}{\text{cross section}} = \frac{\text{constant}}{\sqrt{\text{depth.}}}$$

From experiment the following is derived for field coils of open type machines with normal peripheral speeds,

$$\Delta = \frac{\text{amperes}}{\text{gross cross section}} = \frac{210}{\sqrt{\text{winding depth}}}$$

The winding depths vary from 4 cm. in small machines to 8 cm. about, in large machines. The depth of winding may be taken as 1.75 λ or 3.5 $\log_{10} D$.

AIR GAP A MULTIPLIER.

In determining the ampere turns for the air gap the armature is treated as if the slots were entirely closed.

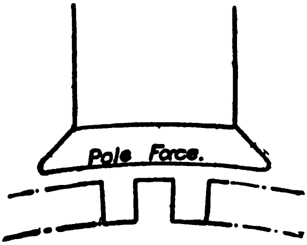

Fig. 22.

It is equivalent to assuming a smooth core armature. The actual flux per pole is employed, and on the assumption as to a smooth core, the ampere turns are calculated for the width of air gap, given by the radial distance from the tops of the teeth to the pole-shoe. To correct for the presence of actual slots a multiplier is used for these ampere turns, which of course should always be greater than unity. From stream line drawings Professor Arnold has deduced certain values for this multiplier which is designated by K_1. The value of K_1 can be represented by the following:

$$K_1 = \frac{1}{r + \sqrt{\frac{\delta}{z}}\,(1 - r)}$$

RATIO OF D TO p IN TERMS OF l.

$$\frac{D}{p} = \varepsilon\, l$$

when ε has the value of $\frac{1}{2}$.

SPECIFIC RESISTANCE OF COPPER.

The specific resistance per cm.^{3} of copper can be assumed as (taking into account permissible temperature rise)

$$\varphi = 2 \times 10^{-6} \text{ ohms.}$$

COPPER FACTOR.

The value of ϕ the copper factor for ordinary armature construction can be taken as .38. The copper factor of shunt fields is about .54.

BEARING FRICTION AND WINDAGE WATTS.

$$\text{Watts} = .000015\, d_s^{2\cdot 5}\, l_s\, R^{1\cdot 5},$$

where d_s = diameter of shaft in the journal in cm. l_s = length of shaft in journal in cm. and R = revs. per minute.

BRUSH FRICTION WATTS.

$$\text{Watts} = .000135\, D_c\, R\, \sigma$$

D_c = diameter of commutator in cm.
σ = area of all brushes in cm.^{2} and
R = revs. per minute.

BRUSH VOLTAGE DROP.

$$\text{Voltage drop} = .149\, \Delta_b = v$$

Δ_b = current density in brush per cm.^{2}

FIELD COIL LENGTH OF MEAN TURN.

The length of mean turn of the field coil can be approximated to in the following way. It can be taken to be a mean between the circumscribing circle of a square formed by the pole-shoe and an inscribed circle in the same square. Since the length of the pole-shoe is $\frac{\pi D f}{p}$ the inscribed circle will measure $\pi . \frac{\pi D f}{p} = \frac{\pi^2 D f}{p}$. The circumscribing circle will measure $\sqrt{2} . \frac{\pi^2 D f}{p}$. The mean between these two circles will therefore be

$$(\tfrac{1}{2} + \tfrac{1}{2}\sqrt{2}) \frac{\pi^2 D f}{p} = 1.2 \frac{\pi^2 D f}{p}$$

Thus the

$$L.M.T = 1.2 \frac{\pi^2 D f}{p}$$

FIELD EXCITATION WATTS FOR A GIVEN **A**.

Let A = cross-section of coil and Λ = length of mean turn. Let ampere turns = **A**. The Joulean loss will therefore be

$$\mathbf{A}^2 \times \frac{\psi}{\phi} . \frac{\Lambda}{A}$$

Now ψ for copper at 40° C is 2×10^{-6} ohms the value of ϕ for shunt fields is .54. Hence Joulean loss is given by

$$\text{watts per bobbin} = (\mathbf{A})^2 . \frac{2 \times 10^{-6}}{.54} . \frac{\Lambda}{A}$$

$$= \left(\frac{\mathbf{A}}{1000}\right)^2 \times \frac{2}{.54} \times \frac{\Lambda}{A}$$

$$= 3.7\left(\frac{\mathbf{A}}{1000}\right)^2 \times \frac{L.M.T}{\text{cross section}}$$

$$= 3.7 \times 10^{-6} \times \frac{\mathbf{A}}{\text{cross section}} \times L.\ M.\ T. \times \mathbf{A}$$

$$\text{But } \frac{\mathbf{A}}{\text{cross section}} = \frac{210}{\sqrt{\text{winding depth}}}$$

$$\text{and } L.M.T = 1.2 \times \frac{\pi^2 D f}{p}$$

Hence watts per bobbin

$$= \frac{3.7 \times 10^{-6} \times 210}{\sqrt{\text{winding depth}}} \times 1.2 \times \pi^2 \times f . \frac{D}{p} \times \mathbf{A}$$

and for the entire machine or p poles it will be

$$\text{Total excitation loss} = \mathbf{A}\,\frac{3.7 \times 10^{-6} \times 210 \times 1.2 \times \pi^2 \times f D}{\sqrt{\text{winding depth}}}$$

The specific excitation loss, or loss per $\pi D l i v$ for the entire p poles is

$$= \frac{3.7 \times 10^{-6} \times 210 \times 1.2 \times \pi^2 f D \mathbf{A}}{\sqrt{\text{winding depth}} \times \pi D l i v}$$

$$= 2.93 \times \frac{f}{l i v} \times \frac{\frac{\mathbf{A}}{1000}}{\sqrt{\text{winding depth}}}$$

and if $f = .7$ and $iv = .72$

$$\textit{Specific excitation loss} = 2.85 \times \frac{\frac{\mathrm{A}}{1000}}{l\sqrt{w.d.}}$$

The winding depth of the fields can be taken as 1.75 λ. Hence

$$s.\,e.\,l = 2.15 \times \frac{\frac{\mathrm{A}}{1000}}{l\sqrt{\lambda}}$$

SPECIFIC OUTPUT.

$$\textit{The specific output} = \frac{W}{\pi D l i v} = \frac{D R S}{\pi \cdot i\, v} = .442\,.\,D R S$$

EQUATIONS FOR $(B\,H)$ CURVE OF MILD STEEL.

From B equal to 7000 to 17000, $\log_{10} H = \dfrac{B/1000}{25.5 - B/1000}$

From B equal to 19000 to 26000, $\log_{10} H = .13 \left(\dfrac{B}{1000}\right)$

TEETH IRON LOSS.

The iron loss can be separated into two losses: Hysteresis depending directly upon the frequency; and the eddy current loss depending upon the square of the frequency. The total iron loss is given by

Iron loss $= (a{\sim}B^x + b{\sim}^2\, B^2)$. volume.

Here x is the index of the maximum induction upon which the hysteresis loss is known to depend. The corresponding index for the eddy loss is the square Transforming,

$$\text{iron loss} = \left(a{\sim}\,\frac{B^x}{B^2} + b\,{\sim}^2\right) B^2 \text{ volume}$$

Now with the densities usually employed in dynamo design and noting the fact that the eddy loss is such an important item of the entire loss, the quantity $\frac{B^x}{B^2}$ can be approximated to by unity. Furthermore for ordinary frequencies $a{\sim}+b{\sim}^2$ can be replaced by a quantity $c{\sim}^\nu$. The loss hence reduces to the simple expression

$$\textit{Iron loss} = c{\sim}^\nu B^2 \times \text{volume}$$

$$= 4.14 \times 10^{-12} {\sim}^{1.4} B^2 \times \text{volume (cm}^3\text{)}.$$

The frequency is of course in cycles per second.

The total iron loss separated into its components gives:
Hysteresis per cycle per second $= 6.85 \times 10^{-12} \times B^2 \times$ volume
Eddy loss per cycle per second $= 3.28 \times 10^{-13} \times B^2 \times$ volume

Specific teeth iron loss $= \alpha\, B^2 {\sim}^x\, r\, \lambda$

$$= 4.14 \times 10^{-4} \left(\frac{B}{10.000}\right)^2 {\sim}^{1.4} . r . \lambda$$

SPECIFIC CORE IRON LOSS.

$$\textit{Specific core iron loss} = \alpha\, \pi^2 \varepsilon\, l \left(1 - \frac{1}{p}\right) B_a^2\, r^2 {\sim}^{1.4}$$

$$= 5.1 \times 10^{-4}\, l \left(1 - \frac{1}{p}\right) \left(\frac{B}{10.000}\right)^2 r^2\, m^2 {\sim}^{1.4}$$

where $B_a = m.B$

SPECIFIC TOTAL IRON LOSS.

$i =$ Specific teeth iron loss

$$\left\{1 + \frac{5.1 \times 10^{-4}\, l \left(1 - \frac{1}{p}\right) \left(\frac{B}{10.000}\right)^2 m^2 r^2 \sim^{1.4}}{4.14 \times 10^{-4} \left(\frac{B}{10.000}\right)^2 \sim^{1.4} r \lambda}\right\}$$

$=$ Specific Teeth iron loss $\left\{ 1 + 1.23\, m^2 r \left(1 - \frac{1}{p}\right) \frac{l}{\lambda} \right\}$

SPECIFIC INCREMENTAL EDDY CURRENT LOSS.

$$n = \frac{222\, \lambda}{r^3 \delta^2 (D\, l)^2} \cdot \left(\frac{W \times 10^{-3}}{B/1000}\right)^2$$

SPECIFIC ARMATURE COPPER LOSS.

Specific armature copper loss

$$= \left(\frac{60 \times 10^8}{\pi^2 f}\right)^2 \left(1 + \frac{2\, \pi\, \varepsilon}{\sqrt{3}}\right) \frac{\psi}{\phi} \cdot \frac{S^2}{(1-r)\, r^2 \lambda} B_a^2\, (iv)^3$$

Taking $f = .7$, $\varepsilon = .5$, $\phi = .38$, $\psi = 2 \times 10^{-6}$, $iv = .72$

$$s.c.l = \frac{.3 \times (S \times 10^3)^2}{(B_a/10000)^2\, \lambda\, r^2\, (1 - r)}$$

FORMULA FOR B.

The current density follows from

$$i_s = \frac{12 \times S \times 10^4}{m\, (B/10.000)\, \lambda\, (1 - r)\, \phi}$$

$$\text{Hence } B = \frac{12 \times S \times 10^4 \times 10000}{m \times i_s \lambda (1-r) \phi}$$

For trial value of B take $i_s = 70$, $(1-r) = .5$, $m = 1.05$ and $\phi = .38$. Then

$$B = \frac{3200 \times S \times 10^4}{\lambda}$$

EMPIRICAL FORMULA FOR AIR GAP.

$$\delta = \frac{D}{D+50}$$

Professor Niethammer gives the following minimum rule from mechanical considerations:

$$\delta > \frac{D}{200 + 2D}$$

FIELD SPARKING VOLTAGES.

$R.V$ = sparking voltage due to field at point of commutation (geometrical neutral position).

$$e_l = 1.31 \frac{E}{\nu} . p . \frac{S \times 10^2}{r^2 (B/1000)^2} < 6$$

Here m has been taken as 1.05, f as .7, and $i\,v$ as .72. The formula is for *lap windings.*

For *wave windings* the following obtains:

$$R\,V = e_w = 2.62 \frac{E}{\nu} . \frac{S \times 10^2}{r^2 (B/10.000)^2} < 6$$

SPECIFIC RADIATIVE CAPACITY.

The axial length of the barrel surface of the armature is

$$l\left(1+\frac{\pi \varepsilon}{\sqrt{3}}\right)$$

Total barrel surface $= \pi D l\left(1+\frac{\pi \varepsilon}{\sqrt{3}}\right)$, hence the specific surface is $\dfrac{\pi D l\left(1+\frac{\pi \varepsilon}{\sqrt{3}}\right)}{\pi D l i v}$. Now $\dfrac{W}{\text{cm}^2 \text{of barrel surface}}$

$= 4.12 \times 10^{-4}\, p.\, s$ (by direct experiment) where $p.\, s. =$ peripheral speed. Hence $\dfrac{W}{\pi D l\left(1+\frac{\pi \varepsilon}{\sqrt{3}}\right)} = 4.12 \times 10^{-4}\, p.s.$

and therefore,

$$\frac{W}{\pi D l i v} = \frac{1+\frac{\pi \varepsilon}{\sqrt{3}}}{i v} .4.12 \times 10^{-4}\, p.s., \text{ or,}$$

Specific radiative capacity $= 1.09 \times 10^{-4}\, p.s.$

CHAPTER XVII.

EXAMPLES OF DESIGN.

To illustrate the foregoing principles the design of three machines will be undertaken; a 10 kw. at 1100 revs., a 100 kw. at 450 revs., and a 1000 kw. at 220 revs. For the $D^2 l$ the formula

$D^2 l.R = 36\times10^4 (1.5+\sqrt{\text{kw.}})^2$ will be used.

In this manner the following is arrived at

Output	Revs.	$D^2 l$
10 kw.	1100	7,100
100 kw.	450	106,000
1000 kw.	220	1,795,000

For determining the number of poles the empirical rule $2^{\frac{p-4}{2}} = \frac{D^2 l}{150\ 000}$ will be taken.

Obviously if $p = 4$ the $D^2 l$ has a maximum value of 150,000. Thus the 10 kw. and the 100 kw. machines are indicated as four pole machines. Taking now $p = 10$, $2^3 = 8$, and thus the $D^2 l$ for eight poles is $8\times150{,}000 = 1{,}200{,}000$. For $p = 12$ the $D^2 l = 2{,}400{,}000$. So that for the 1000 kw. machines the trial values of p are 10 and 12. To determine the diameter and length of a machine the relation $\frac{D}{l} = \varepsilon\, p$ is used where $\varepsilon = .5$.

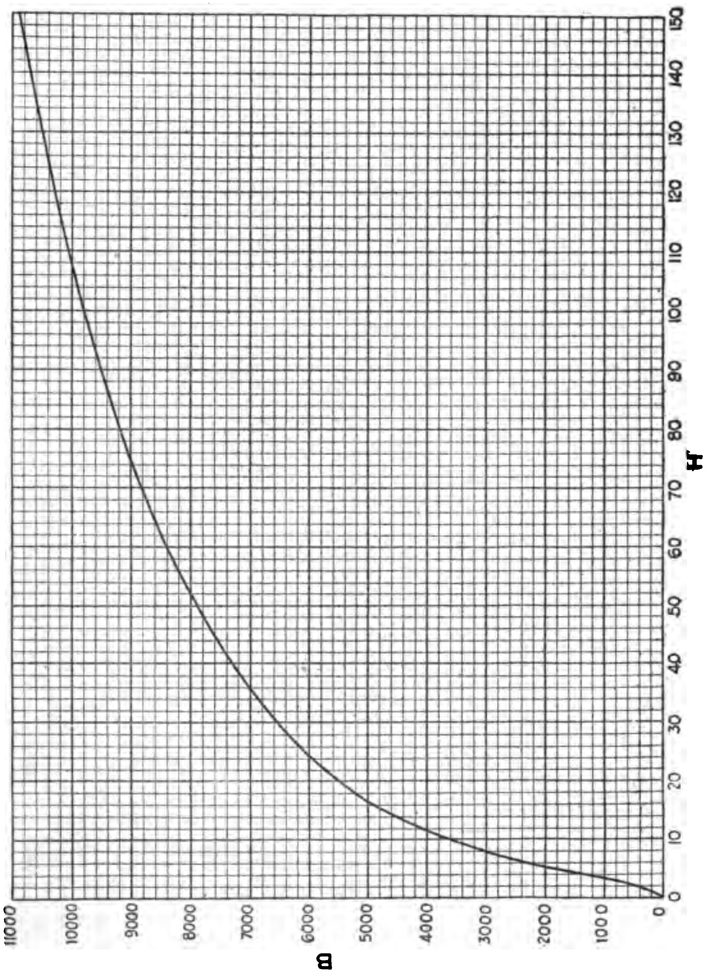

FIG. 23.—Magnetization Curve for Cast Iron.

Thus having given the $D^2\,l$ the following results:

10 kw. at 1100 revs. with 4 poles $D=24$ $l=12$ cm.
100 " " 450 " " 4 " $D=60$ $l=30$ cm.
100 " " 450 " " 6 " $D=75$ $l=25$ cm.
1000 " " 220 " " 10 " $D=210$ $l=42$ cm.

In the 100 kw. machine the influence of the number of poles in affecting the design is desired to be shown. The formula for determining the magnetic density is

$B = \dfrac{3200\times S\times 10^4}{\lambda}$ but in these cases higher trial densities will be employed to show how well the chosen final densities are in agreement with the above formula. Taking the case of the 10 kw. the first step is to obtain the trial magnetic density, then with the value of B to evaluate the specific iron and copper losses till only the value of r remains. The depth of slot is given by the empirical formula $\lambda = 2\log_{10} D$. Having a value of $D = 24$ it follows that $\lambda = 2.76$ cm. Designating the specific teeth iron loss by t.

$$t = 4.14\times 10^{-4}\left(\frac{B}{10,000}\right)^2 \sim^{1.4} r\,\lambda$$

Also designating by c the specific copper loss

$$c = \frac{.3\times(S\times 10^3)^2}{m^2\, r^2\,\lambda\,(1-r)\left(\dfrac{B}{10,000}\right)^2}$$

With $p = 4$, $D = 24$, $l = 12$, $\delta = .325$,* $\lambda = 2.76$,

---

*In this machine δ could be taken equal to $\frac{24}{248} = 1$ millimetre by Niethammer's formula.

$S \times 10^3 = 1.28$, and $R = 1100$. It follows that $\sim = 36\frac{2}{3} = \frac{R p}{120}$, $\sim^{1.4} = 155$, and that

$$c = \frac{.072}{r^2\,(1-r)}$$

with $m = 1.05$ and $B = 15000$.

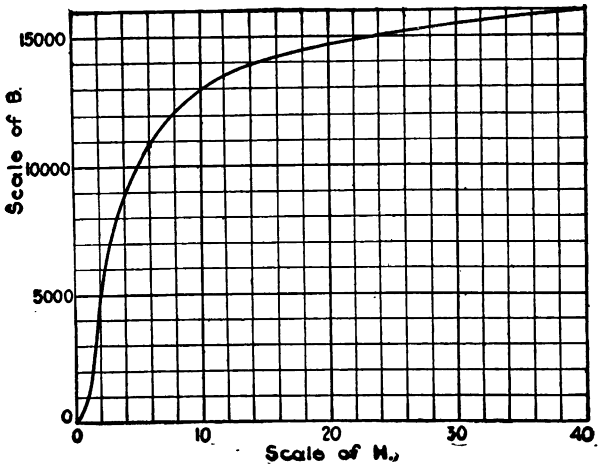

FIG. 24.—Magnetization Curve for Soft Iron.

If i = specific total core loss, i

$$= t\left\{1 + 1.23 m^2 r\left(1 - \frac{1}{p}\right)\frac{l}{\lambda}\right\}$$ from the formulary.

Thus finally

$$i = .398\,r\,[1 + 4.43\,r]$$

$$c = \frac{.072}{r^2\,(1-r)}, \text{ and}$$

$$\frac{W}{\text{sp. cm}^2} = 1.09 \times 1.38 = 1.505$$

With these three formulæ the total losses that can be generated per cm^2 of $\pi D l i v$ can be gone into for different values of r. The maximum permissible generation for a temperature rise of 40° C. is 1.505 obtained above. Tabulating, the result is as follows:

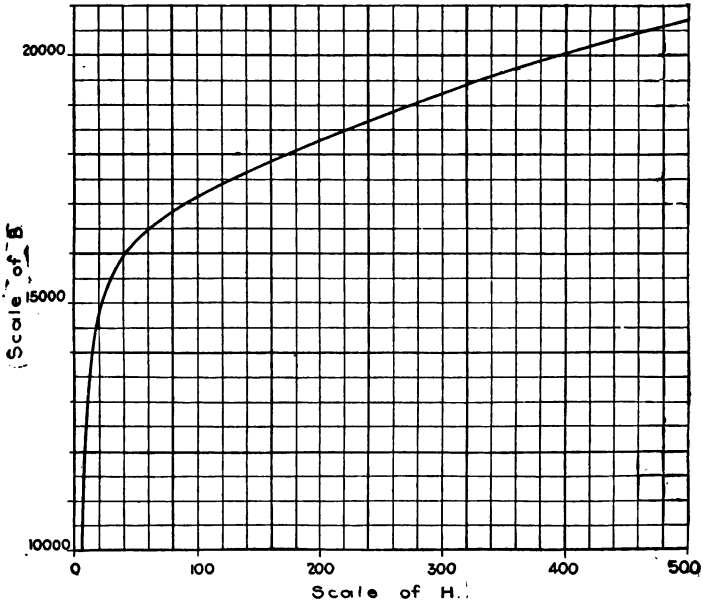

FIG. 25.—Magnetization Curve for Soft Iron.

$r =$	.4	.5	.6	.7
$i =$	.442	.602	.874	1.14
$c =$	.75	.575	.5	.49
$i+c$	1.192	1.177	1.374	1.63

These values are for a density of 15,000 B. By a theorem already explained the minimum values for $i+c$, changing the density, is obtained by taking $2\sqrt{i \times c}$, where the value of the new B' is $\frac{B}{x^{\frac{1}{2}}}$, and where $x = \frac{i}{c}$. In this manner the values are as follows:

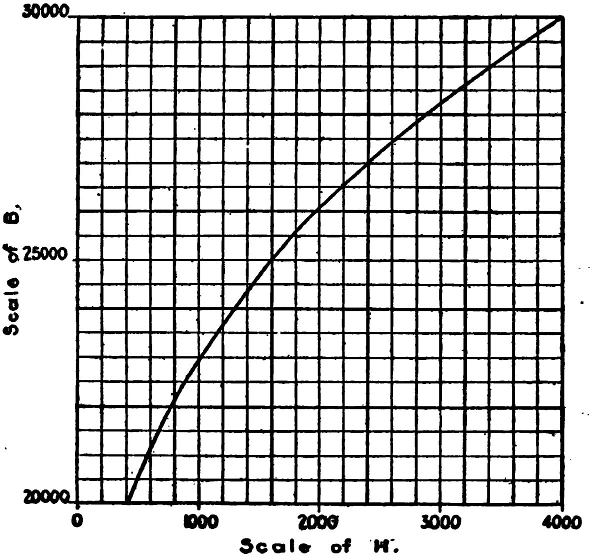

FIG. 26.

r	=	.4	.5	.6	.7
$2\sqrt{i \times c}$	=	1.15	1.176	1.32	1 495
x	=	.589	1.05	1 75	2.32
B'	=	18,150	14,800	13,050	12,150

The specific excitation loss for the teeth and air gap is generally given by the formula

$$\text{Specific excitation loss} = \frac{2.93 \times f}{l\,i\,v} \cdot \frac{\frac{\mathbf{A}}{1000}}{\sqrt{175\lambda}}$$

Making the necessary substitutions,

$$s.e.\,l = .108\,\frac{\mathbf{A}}{1000}$$

Furthermore the formula for current density is

$$i_s = \frac{12 \times S \times 10^4}{m \times \lambda\,(1-r)\left(\frac{B}{10{,}000}\right)}$$

Performing the necessary calculations, knowing the new values of B', and determining the Arnold multiplier K_1 with $\frac{z}{\delta} = \frac{2.24}{.325} = 6.9$, the specific teeth and air gap **A** can be also tabulated. The value of z the tooth pitch is given by $z = 2 + \frac{D}{100}$. The value of K_1 is then read off from Fig. 27. In this way, one has finally

r	=	.4	.5	.6	.7
$2\sqrt{i \times c}$	=	1.15	1.176	1.32	1.495
$s.e.l$	=	.846	.807	.849	.92
$i+c+e$*	=	1.996	1.983	2.169	2.415
B'	=	18,150	14,800	13,050	12,150
A air gap and teeth	=	2.960	2.530	2.430	2.440
i_s	=	48.6	71.6	102.	

---

* e is taken to represent the specific excitation loss.

The particular magnetic density finally decided on is $B' = 15{,}000$ with $r = .5$ and $\lambda = 2.76$. This is done because, as is obvious from the table, the temperature rise will be quite low (permitting of a great overload), and further the efficiency will be also better. The current density in the armature copper will be 71.6.

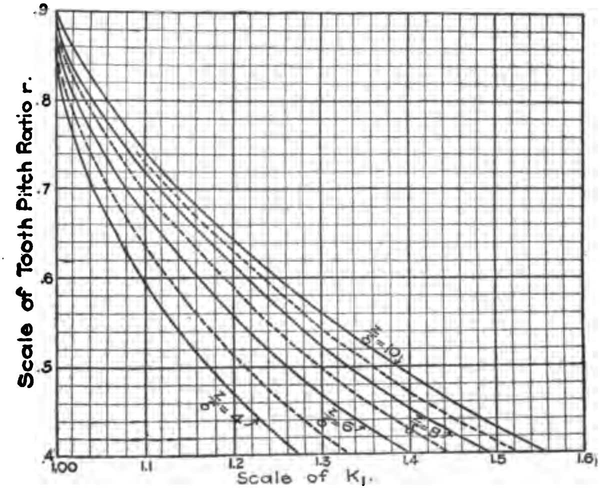

FIG. 27.—Air Gap Ampere Turn Correction Curves

Attention will now be turned toward designing the commutator for this machine. The formula for the reaction voltage is

$$e_w = < 6 = 2.62\,\frac{E}{\nu}\,\cdot\,\frac{S \times 10^2}{r^2(B/10{,}000)^2}$$

for a wave wound machine.

Assume a maximum voltage of 600 volts, then ν = number of commutator segments.

$$= \frac{600 \times 2.62 \times .128}{6 \times .25 \times 2.25} = 57.7$$

where $r = .5$ and $B = 15{,}000$.

Taking a minimum commutator bar width of 1 cm. the diameter of the commutator must be

$$\frac{55.7}{\pi} = D_c = 17.75 \text{ or say } 18 \text{ cm.}$$

A formula had previously been developed for the current density in the brush with a view to maximum economy in the loss.

$$\Delta_b = \sqrt{\frac{D_c R}{1100}}$$

Thus $\Delta_b = \sqrt{\frac{18 \times 1100}{1100}} = 4.25$ amperes per cm^2. Since the output is 10 kw. the amperage must be 16.7 and thus the cross section of brush is $\frac{1}{2} \times \frac{16.7}{4.25} = 1.965$ cm^2. The following rule for width of brush may be taken that is, that the brush should not cover more than .09 of the polar pitch of the commutator. From this it follows since the diameter of the commutator is 18 cm. that the width is $\frac{.09 \times \pi \times 18}{4}$ or 1.25 centimetres. With this the active length of commutator follows which is $\frac{1.965}{1.25} = 1.57$. Bv taking a minimum voltage of 125 volts the current will be increased fourfold and hence the active length of commutator should be $4 \times 1.57 = 6.3$ cm. The minimum width of brush is one centimetre.

Turning now to the 100 kw. machine two designs will be investigated; one for six poles, and the other for four poles. The first step is to obtain the formulæ for iron and copper loss, using the empirical value of B, and with the depth of slot λ given by $\lambda = 2 \log_{10} D$. In this case $\lambda = 3.75$ for the six polar. The value of δ is by $\delta = \frac{D}{D+50} = .6$ cm. Thus with $B = 21\,200$

$$i = .548\, r\,[1 + 7.55\, r]$$

$$c = \frac{.0712}{r^2\,(1-r)}$$

The permissible radiating watts for the armature per $\pi D l i v$. is $1.925 = 1.09 \times 10^{-3}\, p.s.$

Constructing a table there is obtained the following:

r	.4	.5	.6	.7
i	.882	1.31	1.82	2.41
c	.742	.57	.495	.484
$2\sqrt{i.c}$	1.618	1.725	1.9	2.16
$i+c$	1.624	1.88	2.315	2.894
$x = \frac{}{c}$	1.19	2.3	3.68	4.97
$B' = \frac{21200}{x^{\frac{1}{4}}}$	20,300	17,250	15,300	14,200
H'	450	105	30	15
$B'r$	8120	8625	9180	9940
$H'(1-r)$	270	53	12	5
K'	1.3	1.2	1.11	1.06
$\frac{10}{4\pi}K'\delta[B'r+H(1-r)]$	5230	4980	4860	5060
$\frac{10}{4\pi}H'\lambda$	1350	315	90	45
Total **A**, teeth and air gap	6580	5295	4950	5105

Excitation loss $e = .0444 \frac{\mathbf{A}}{1000}$

e	.292	.235	.220	.227
$i'+c'$	1.618	1.725	1.9	2.16
i_s	54.6	74	104.5	

The choice is therefore taken of $B = 17.250$, and $r = .5$.

Turning to the design of the commutator with a voltage of 600 and a lap winding, from the R. V. formula

$$\nu = \frac{2.62 \times 600 \times .21 \times 3}{6 \times 1.725^2 \times .25} = 222$$

With a maximum number of 222 one centimetre commutator parts, the diameter will be $D_c = \frac{222}{\pi} = 70.$

The current density in the brushes will follow from

$$\Delta_b = \sqrt{\frac{D_c \times R}{1100}} = \sqrt{\frac{70 \times 450}{1100}} = 5.35$$

With a maximum amperage of $\frac{100.000}{3 \times 125} = 270$ (taking a minimum voltage of 125 V.) the area of the brushes must be $\frac{270}{5.25} = 51.4$. Taking the width of brush $= .09 \times \frac{70 \times \pi}{6} = 3.3$. Hence the active length of commumutator necessary is $\frac{51.4}{3.3} = 15.5$ cm.

Investigating the four-pole 100-kw. machine, the trial density will be 22,300 and $\lambda = 3.55$. In this manner

$$i = .326\, r\, [1 + 5.72\, r]$$

$$c = \frac{.069}{r^2\,(1 - r)} \text{ and}$$

$$\frac{W}{\text{specific cm}^2} = 1.09 \times p.s. \times 10^{-3} = 1.54$$

Tabulating:

r	.4	.5	.6	.7
i	.429	.628	.869	1.14
c	.72	.553	.48	.47
$2\sqrt{i.c}$	1.11	1.178	1.29	1.465
$i+c$	1.149	1.181	1.349	1.61
$x = \frac{i}{c}$	.596	1.136	1.81	2.43
$B' = \frac{22300}{x^{\frac{1}{4}}}$	26,200	20,600	18,400	17,100
H'	2100	500	200	100
$B'r$	10,500	10,300	11,040	11,970
$H'(1-r)$	1260	250	80	30
K'	1.32	1.2	1.11	1.06
$.8\,\delta K'\,[B'r+H'(1-r)]$	6750	5500	5360	5540
$.8\,H'$	5960;	1420	569	284
Total **A**	12,710	6920	5929	5824
Excitation loss = .057 **A**/1000				
e	.724	.394	.338	.332
i_s	43	65.7	91.8	132

The choice is therefore B = 20,600 and r = .5. Investigating for the commutator, with E = 600 and a lap winding, the maximum necessary commutator parts are

$$\nu = \frac{2.62 \times 600 \times .21 \times 2}{6 \times 2.06^2 \times .25} = 104$$

Thus the diameter of the commutator D_c would be $\frac{104}{\pi} = 33$. This would give a ratio of diameter of commutator to diameter of core $= .55 = \frac{33}{60}$ This is rather small for a machine of this size. For the 10 kw. machine the ratio was .7 and for a 1000 kw. it would be about .85. Hence in this case it may be taken at .75 which would give a commutator diameter of 40 cm. The value of $\Delta_b = \sqrt{\frac{40 \times 450}{1100}} = 4.25$. (With a diameter

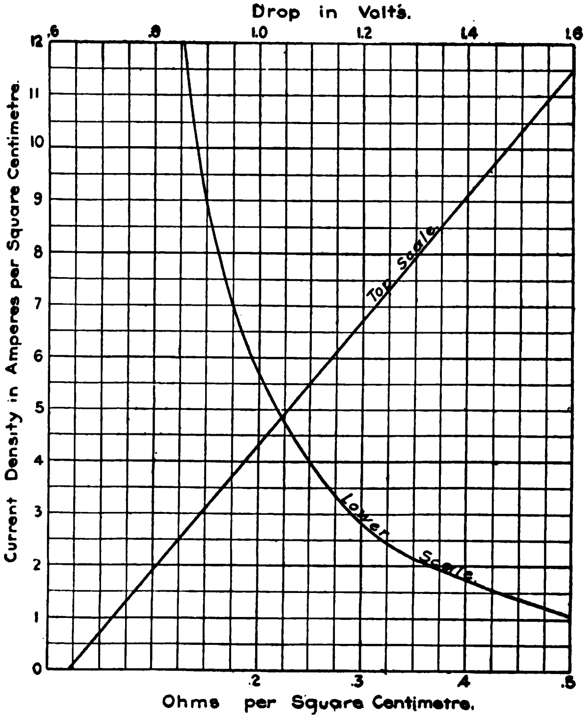

FIG. 28.—Resistance and Voltage Drop Curves for Carbon Brushes at Average Pressures and Speeds.

of 33 cm. the brush density would be quite low.) The width of brush is $\frac{\pi \times 40 \times .09}{4} = 2.83$ or say 2.75 which is rather large for a carbon brush. Since the output is 100 kw., at 125 volts the amperage is 400 and thus the active length of commutator is $\frac{400}{4.25 \times 2.75} = 34$ cm. A higher density could be employed and then the active length of commutator would be reduced to 29 cm.

For the 1000 kw. machine with $p = 10$ the trial magnetic density will be 25,000 and for $\lambda = 4.65$ with $l = 42$ and $D = 210$

$$i = .685\,r\,(1+11\,r)$$

$$c = \frac{.0684}{r^2\,(1-r)}$$

The following table represents the calculation based on these formulæ. The watts that can be radiated per $\pi\,D\,l\,i\,v$ is $1.09 \times 2.42 = 2.62$.

r	.4	.5	.6	.7
i	1.48	2.23	3.12	4.17
c	.713	.546	.475	.465
$i+c$	2.193	2.776	3.595	4.735
$2\sqrt{i.c}$	2.06	2.21	2.44	2.79
$x = \frac{i}{c}$	2.08	4.08	6.55	8.88
B'	20,800	17,600	15,600	14,500
H'	500	135	40	12
$B'\,r$	8320	8800	9360	10,150
$H'\,(1-r)$	300	68	16	4
K'	1.33	1.21	1.13	1.06
$.8\,\delta\,K'\,[(B'r)\,H'(1-r)]$	7400	6910	6840	6950
$.8\,H'\,\lambda$	1860	249	595	15
Excitation loss = .0238 **A**/1000				
e	.22	.170	.164	.166
$2\sqrt{i\,c}$	2.06	2.21	2.44	2.79
i_s	53.2	75.5	106.5	

The choice therefore is $B = 17{,}600$, $r = .5$. For the commutator the maximum voltage will be 750, and thus

$$\nu = \frac{2.62 \times 750 \times .27 \times 5}{6 \times 1.76^2 \times .25} = 570$$

Thus $D_c = \frac{570}{\pi} = 180$ and $\frac{D_c}{D} = \frac{180}{210} = .85$. The value of $\Delta_b = \sqrt{\frac{180 \times 220}{1100}} = 6$. The brush width is $\frac{.09 \times 180 \times \pi}{10} = 5.1$ and therefore the active length of the commutator would be

$$\frac{1\,000\,000}{5 \times 125 \times 6 \times 5.1} = 52$$

In the foregoing designs no note was taken of the influence of the incremental eddy current loss in the armature as the machine would take on load. Practically the increment will affect the magnetic density and also influence the size of the gap. When the increment is negligable the Niethammer formula for air gap can be acceeded to. The following shows the influence of δ_1. The excitation varies as $a\,\delta_1$ and the increment as $\frac{b}{\delta^2_1}$. Thus for minimum losses with $\frac{d}{d\,\delta'} = 0$

$$a\,\delta_1 = \frac{2\,b}{\delta^2_1}$$

Therefore that value of the air-gap depth is the best, for which, the excitation loss equals twice the value of the incremental iron loss.

If then the old air gap is δ the new air gap should be $\delta' = \left(\frac{2}{x}\right)^{\frac{3}{4}} \delta.$

The following table has reference to the 1000 kw. machine with trial B = 25,000 with δ = .80 8.

r	.4	.5	.6	.7
i	1.48	2.23	3.12	4 17
c	.713	.546	.475	.465
n = incremental loss = $\frac{.0324}{r^3}$				
n	.505	259	.15	.095
$c+n$	1.218	.805	.625	.56
$2\sqrt{i.(c+n)}$	2.68	2.68	2.8	3.06
$x = \frac{i}{c+n}$	1.22	2.77	5.	7.45
$B' = \frac{25000}{x^{\frac{1}{4}}}$	23,800	19,350	16,650	15,100
n'	.566	.433	.338	.26
H'	1250	425	75	27.5
$B'r$	9520	9675	9990	10,570
$H'(1-r)$	750	212	30	8
$B'r+H'(1-r)$	10,270	9887	10,020	10,578
K_1	1.33	1.21	1.13	1.06
$.8 K_1 \delta [B'r+H'(1-r)]$	8800	7740	7300	7250
$.8 H' \lambda$	4650	1580	279	103
Total **A**	13,450	9320	7579	7353
Excitation loss = .0238 **A**/1000				
e	.32	.222	.181	.175
e_δ*	.21	.184	.174	.1725
$x' = \frac{e_\delta}{n'}$	.37	.426	.515	.664

*e_δ signifies the excitation loss for air gap alone.

(Table continued from page 98.)

$\delta' = \left(\frac{2}{x'}\right)^{\frac{1}{3}}\delta$......	1.415	1.31	1.27	1.166
n''	.184	.164	.137	.125
$n'' - n'$...........	.382	.269	.201	.135
$2\sqrt{i(c+n)} - n'' - n'$ —.	2.298	2.411	2.599	2.925
$i_s = \frac{66.3}{(B/10{,}000)(1-r)}$..	46.5	69.	100	

The choice therefore is $B = 19{,}350$, $r = .5$, and $\delta = 1.31$. It was seen that larger magnetic densities diminish the armature self-induction. The design of the commutator would be slightly affected.

With an average time load much below full load the air gap correction would be correspondingly smaller. Thus supposing the time-load factor to be a, it is obvious that the incremental iron loss will be $n \equiv a^2$, but $x = \frac{e}{n}$ and since $n \equiv (W)^2$ and the corrected air gap is $\delta' \equiv \frac{\delta}{x^{\frac{1}{3}}}$ (when δ is the original air gap), the final value of the air gap with load factor a must be $\delta_a' = a^{\frac{2}{3}} \cdot \frac{\delta}{x^{\frac{1}{3}}}$. Thus

$$\delta_a' = a^{\frac{2}{3}}\,\delta'$$

For $a = .75$ and $\delta' = 1.31$, $\delta_a' = 1.08$ cm., and for $a = .5$ with $\delta' = 1.31$ cm., $\delta_a' = {\cdot}826$ cm.

In the matter of choice of poles the following price investigation will be of interest. It is obvious that the losses are increased for an increase in the number of poles. However the armature copper is diminished for an increase in the number of poles and the magnet frame volume is diminished and consequently its cost. Let the cost equation be $a\,D^{x}\,l^{y}$ where D = diameter of armature and l = length. Adopting the equation $D^2\,l = \kappa$ = constant, then cost $= a\,\kappa^{y}\,D^{x-2y}$, which if for a given $D^2\,l$ the cost should be greater for the greater diameters, x must be greater than $2\,y$. Adopting the further condition that $\frac{D}{l} = \varepsilon\,p$ the cost per unit of radiative surface of the armature is proportional to $p\,\frac{x-2\,y+1}{3}\,\kappa\,\frac{x+y-2}{3}$. Obviously if the cost of radiative surface is to be independent of the $D^2\,l$, $x+y-2$ must be equal to zero and x is less than 2. In the same way by substituting in the value for p, y must be less than 1. Thus the formula for cost may be taken as price $= a\,D^{1\cdot5}\,l^{\cdot5}$.

MAGNET FRAME DENSITIES.

It will be found that for poles made of soft steel, $B = 15{,}000$ will be in the neighborhood of the most economical working magnetic density. The condition governing such density will be the total cost of iron and copper. A small magnetic density will necessitate a field coil of large diameter whereas too high a density in the pole would mean an inordinate increase in the field copper. For the yoke when made of cast iron the economic density will be about 7000 B. It may be remarked that with a flux density of 7000 in cast iron there will be required the same magnetizing force as for a density of 15,000 in soft steel.

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